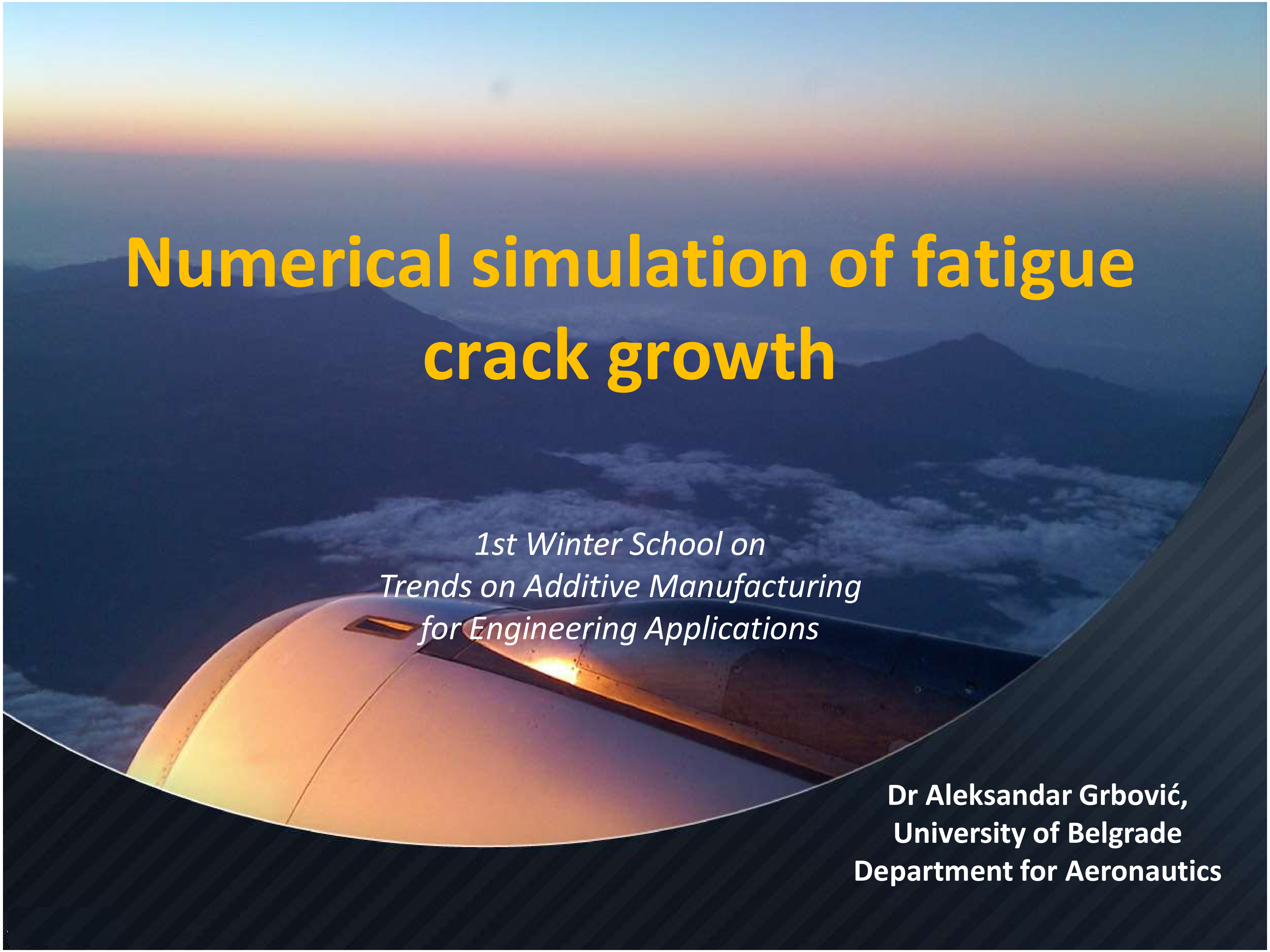


Numerical simulation of fatigue crack growth

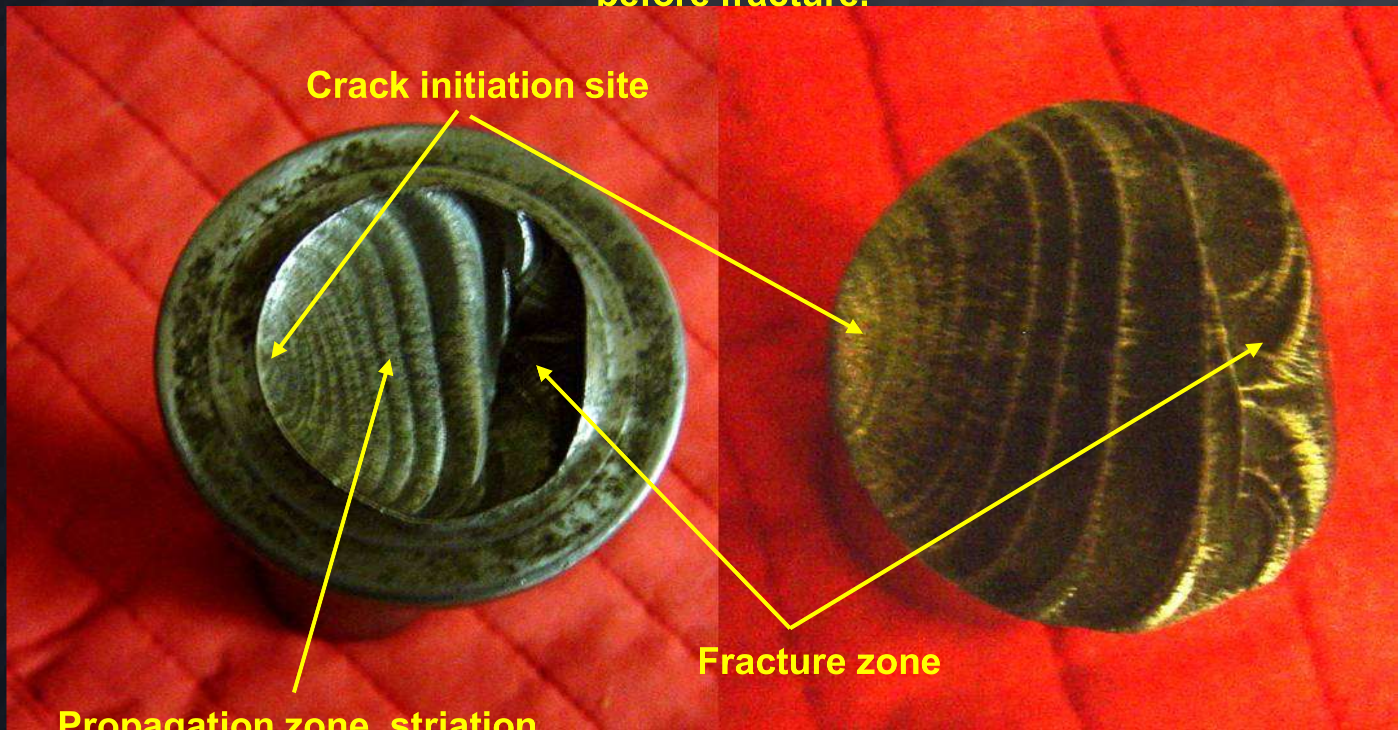
1st Winter School on
Trends on Additive Manufacturing
for Engineering Applications

Dr Aleksandar Grbović,
University of Belgrade
Department for Aeronautics

Material fatigue

- *Fractures due to dynamic loads* have been occurring in load-bearing structures since ancient time, but the study of fracture has intensified when metals became the dominant material in the construction of structures.
- During the 20th century, we learned that repeated loads lead to a process called *material fatigue*.
- **Def**: *Fatigue* is the weakening of a material caused by cyclic loading that results in progressive and localized structural damage and the growth of cracks.

Material fatigue example

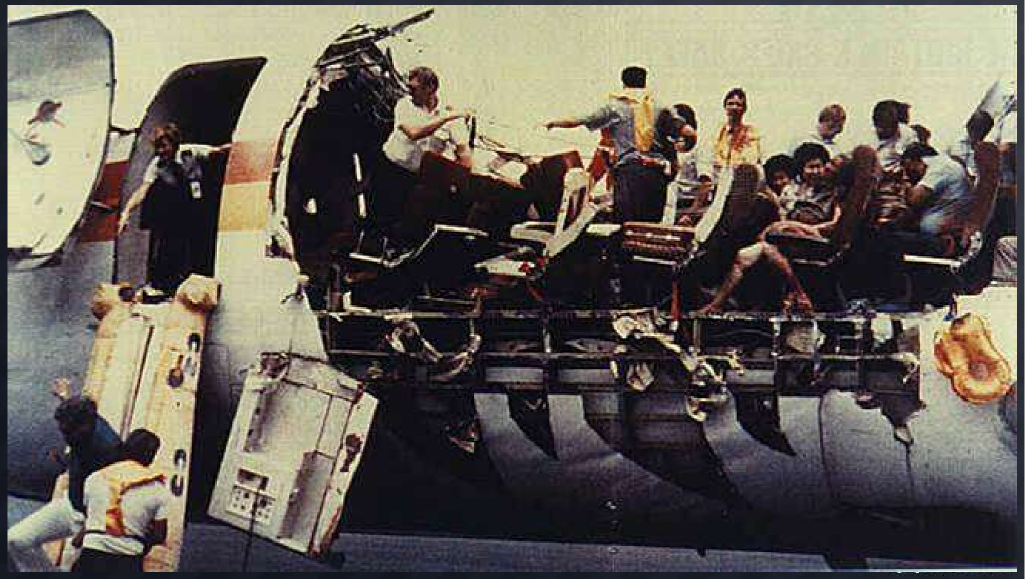


Safe-life and fail-safe approach

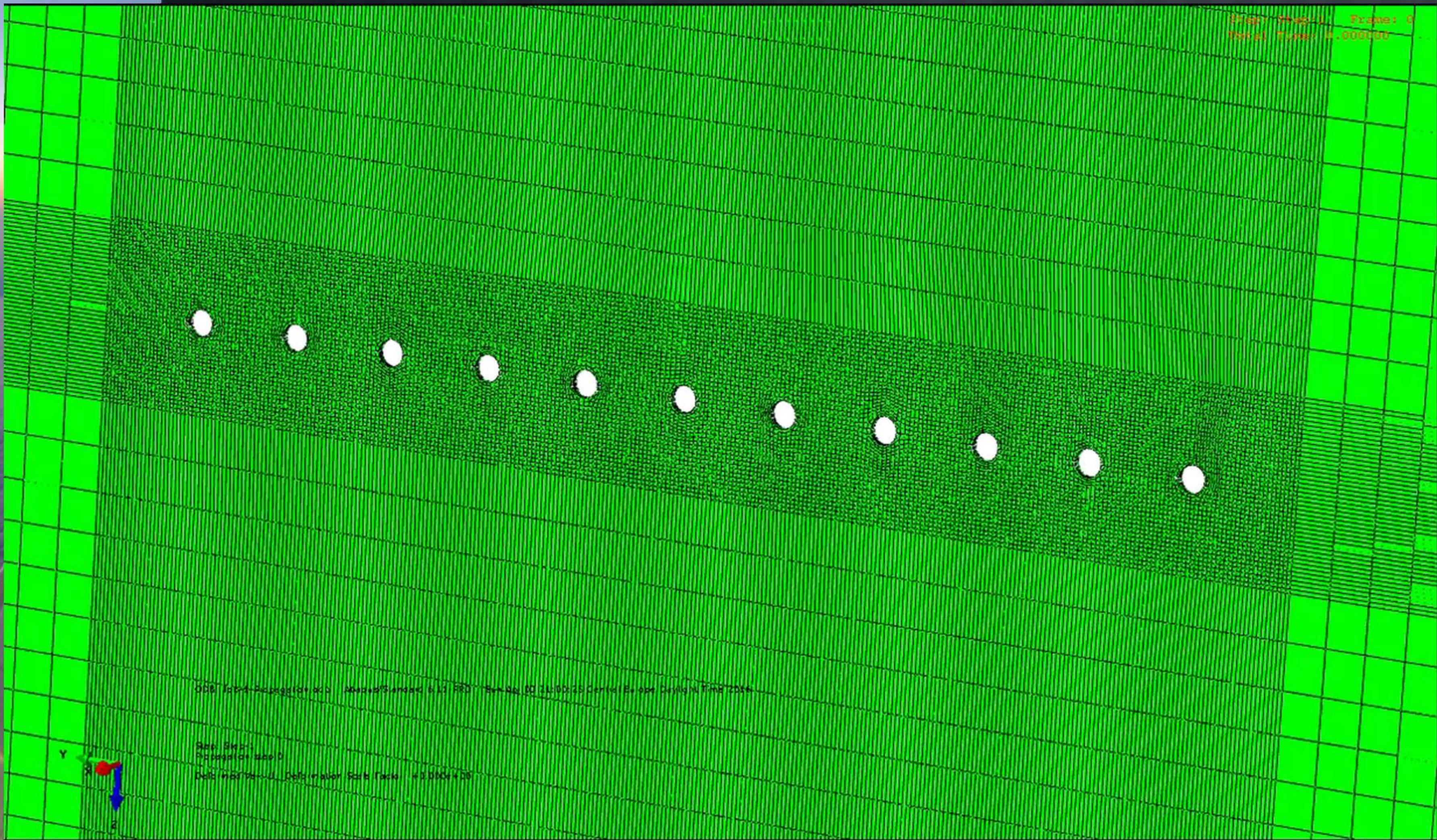
- The conventional approach in design of the fatigue-resistant structure is based on the evaluation of the weakest component's life. This approach is known as *safe-life* and does not take the crack growth into account. At the end of the safe operational life, the component is automatically retired from service (landing gear, wing-fuselage attachment, engine mount, etc.)
- The *fail-safe concept*, on the other hand, is based on the argument that even if an individual member of a large structure fails, there should be sufficient structural integrity in the remaining parts (*structural redundancy*) to enable the structure to operate safely until the damage is detected and repaired (wings, fuselage, engine covers, etc.)

Bad fail-safe design

Hawaii, Aloha Flight 243, Boeing 737, an upper part of the plane's cabin area rips off in mid-flight. Metal fatigue was the cause of the failure.

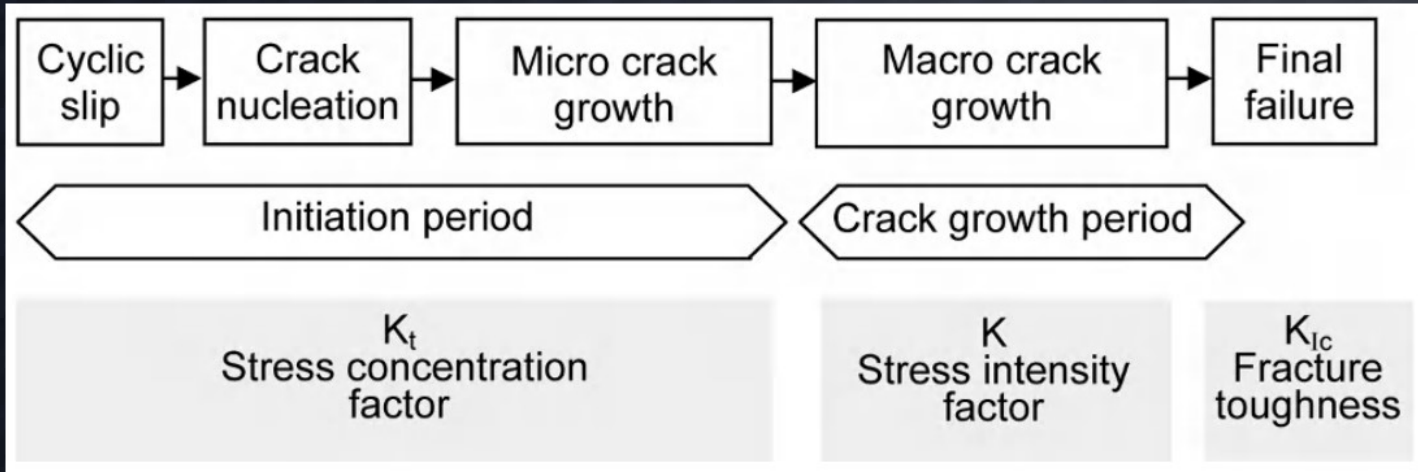


Multiple crack growth in the fuselage skin (XFEM)



Different phases of the fatigue life

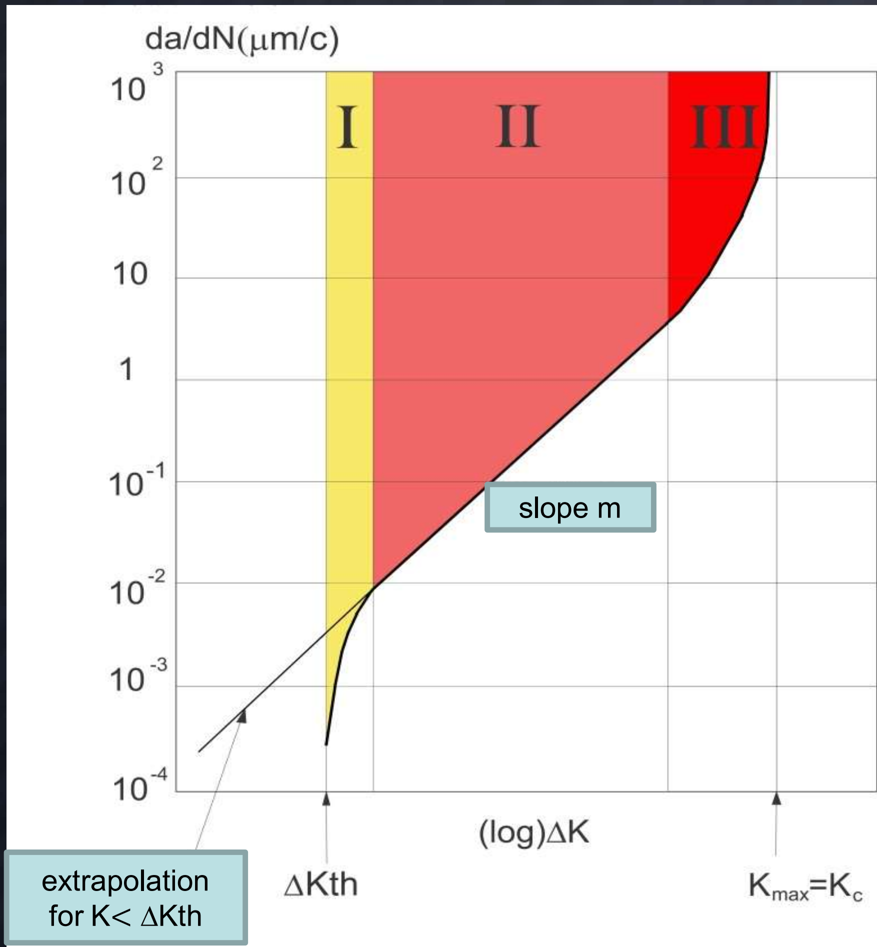
- The fatigue life until failure consists of two periods: *the crack initiation period* and *the crack growth period*.



- The *stress intensity factor (SIF) K* is used for predictions on crack growth. The SIF is a measure of the singular stress term occurring near the tip.

Image taken from: *Fatigue of Structures and Materials*, J. Schijve (2009)

Regions of the crack growth rate



(I) region where ΔK is less than ΔK_{th}, (II) Paris region i (III) fast crack growth region

Equations for region II

- $$\frac{da}{dN} = C[\Delta K]^m$$ Paris equation
- $$\frac{da}{dN} = \frac{C[\Delta K]^m}{\left((1-R)K_C - \Delta K\right)}$$ Forman equation
- $$\frac{da}{dN} = C_0\left(\frac{1}{(1-R)^{1-\gamma}}\Delta K\right)^m$$ Walker equation

$\boldsymbol{R}$ is the stress ratio, ΔK is the stress intensity range, and m is the slope on a log – log scale. The value γ is a material constant.

The value C_0 is the intercept constant *C* for the case where *R = 0*.

- The Paris equation does not account for the stress ratio R.

NASGRO equation for region II

- $$\frac{da}{dN} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^m \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{max}}{K_C}\right)^q}$$

NASGRO equation

where:

$$f = \frac{K_{op}}{K_{max}} = \begin{cases} max(R, A_0 + A_1R + A_2R^2 + A_3R^3), & R \geq 0 \\ A_0 + A_1R, & -1 \leq R < 0 \end{cases}$$

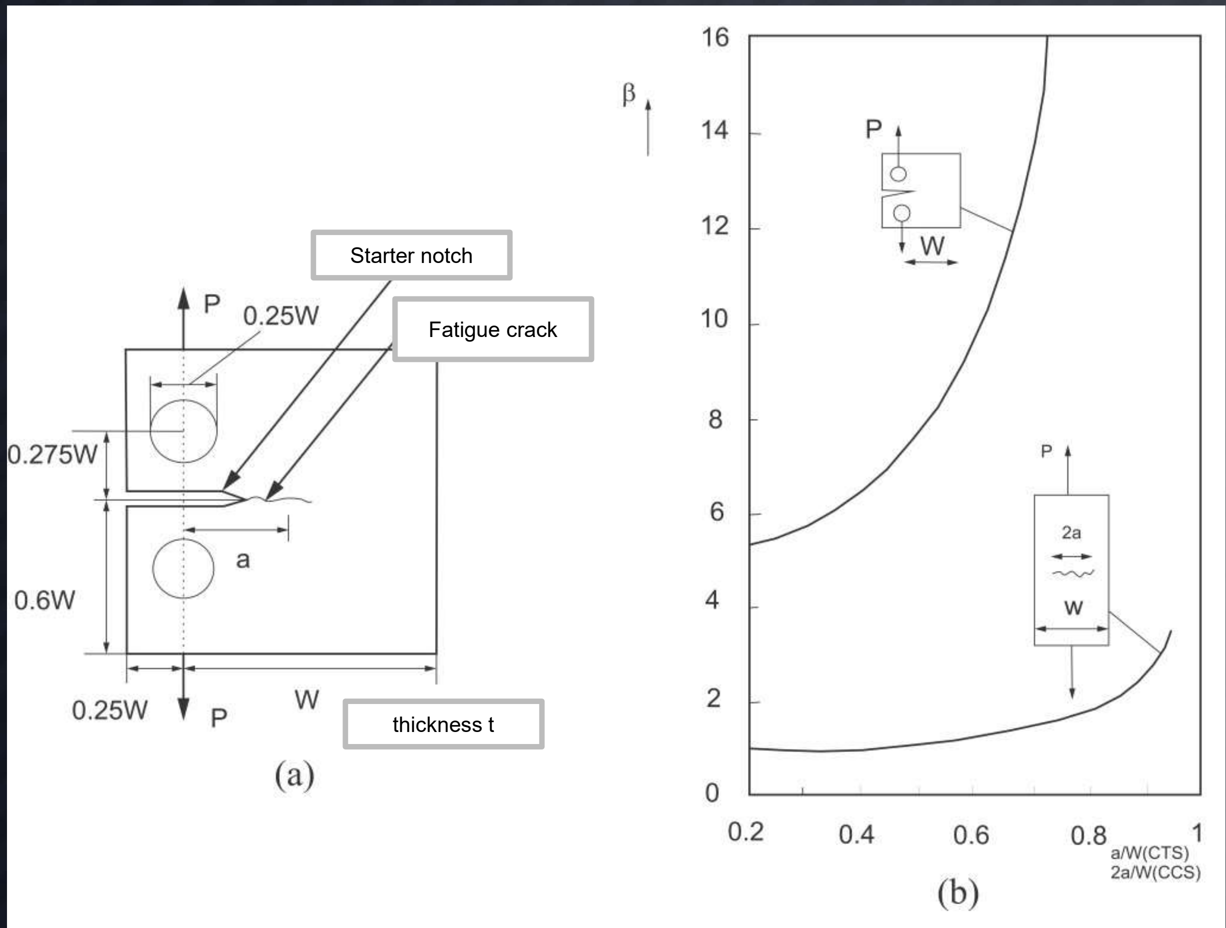
$$A_0 = (0{,}825 - 0{,}34\alpha + 0{,}05\alpha^2) \cdot \left[cos\left(\frac{\frac{\pi}{2}\sigma_{max}}{\sigma_0}\right)\right]^{\frac{1}{\alpha}}$$

$$A_1 = 0(,415 - 0{,}071\alpha) \cdot \frac{\sigma_{max}}{\sigma_0} \qquad A_2 = 1 - A_0 - A_1 - A_3 \qquad A_3 = 2A_0 + A_1 - 1$$

$$\Delta K_{th} = \frac{\Delta K_0\sqrt{\frac{a}{a+a_0}}}{\left(\frac{1-f}{(1-A_0)\cdot(1-R)}\right)^{1+C_{th}R}} \qquad \frac{K_C}{K_{IC}} = 1 + B_K e^{-\left(A_K\frac{t}{t_0}\right)^2} \qquad t_0 = 2{,}5 \cdot \left(K_{IC}/\sigma_{ys}\right)^2$$

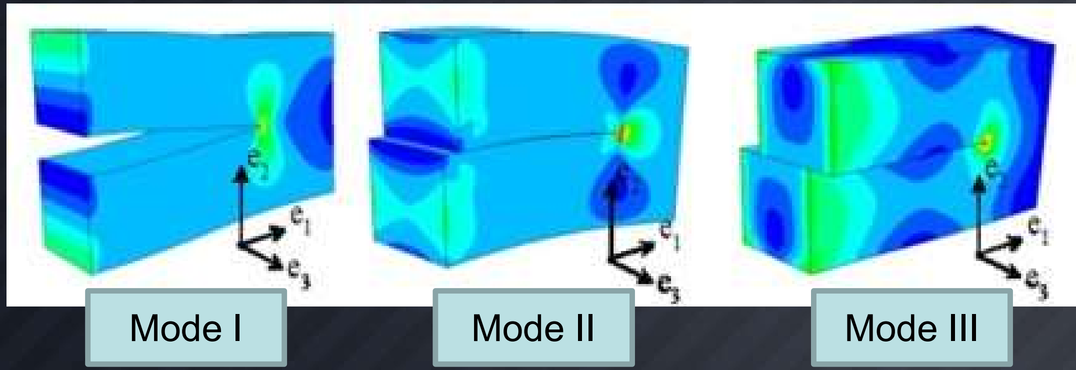
Coefficients C, p, q and n are empirically obtained.

Test specimens for evaluation of K



CT specimen (a) comparison to *CCT* specimen (b)

Stress intensity factor calculation



$$K_I = \lim_{r \to 0} \sqrt{2\pi r}\,\sigma_{22}\ ,\ K_{II} = \lim_{r \to 0} \sqrt{2\pi r}\,\sigma_{12}\ ,\ K_{III} = \lim_{r \to 0} \sqrt{2\pi r}\,\sigma_{32}$$

- Stress state near the crack tip:

angular functions

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}cos\frac{\theta}{2} - \frac{1}{4}cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}sin\frac{\theta}{2} + \frac{3}{4}sin\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}cos\frac{\theta}{2} + \frac{1}{4}cos\frac{3\theta}{2}\right) - \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{3}{4}sin\frac{\theta}{2} + \frac{3}{4}sin\frac{3\theta}{2}\right)$$

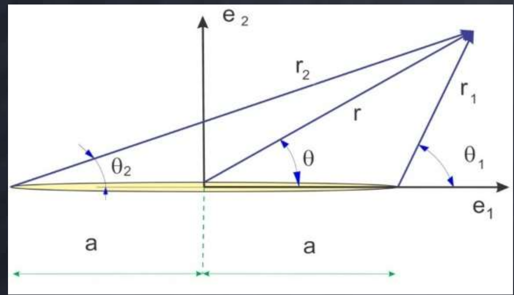
$$\sigma_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}sin\frac{\theta}{2} + \frac{1}{4}sin\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}cos\frac{\theta}{2} + \frac{3}{4}cos\frac{3\theta}{2}\right)$$

Analitical solution for K_I

The stress intensity factors K_I and K_{II} are expressed as:

$$K_I = \sigma' \sqrt{\pi a}\, F_{\mathrm{I}}(a/W) \qquad\qquad K_{II} = \tau' \sqrt{\pi a}\, F_{\mathrm{II}}(a/W)$$

where *a* is the crack length, *W* is the width of the component, and σ', τ' are characteristic stresses in the component.



$$\sigma_{22} = \frac{\sigma_{22}^{\infty} r}{\sqrt{r_1 r_2}}\left[\cos\left(\theta - \frac{\theta_1}{2} - \frac{\theta_2}{2}\right) + \frac{a^2}{r_1 r_2} sin\theta\; sin\frac{3(\theta_1+\theta_2)}{2}\right] + \frac{\sigma_{12}^{\infty} r}{\sqrt{r_1 r_2}}\frac{a^2}{r_1 r_2} sin\theta\; cos\frac{3(\theta_1+\theta_2)}{2}$$

$$u_1 = \frac{(1+\nu)\sigma_{22}^{\infty}\sqrt{r_1 r_2}}{4E}\left[4(1-2\nu)\cos\frac{\theta_1+\theta_2}{2} - \frac{4r(1-\nu)}{\sqrt{r_1 r_2}} cos\theta - \frac{2r^2}{r_1 r_2}(\cos\frac{\theta_1+\theta_2}{2} - \cos\left(2\theta - \frac{\theta_1}{2} - \frac{\theta_2}{2}\right))\right] + \frac{(1+\nu)\sigma_{12}^{\infty}\sqrt{r_1 r_2}}{E}\left[2(1-\nu)\sin\frac{\theta_1+\theta_2}{2} - \frac{2r(1-\nu)}{\sqrt{r_1 r_2}}\sin\theta + \frac{r^2}{r_1 r_2} sin\cos\left(\theta - \frac{\theta_1}{2} - \frac{\theta_2}{2}\right))\right]$$

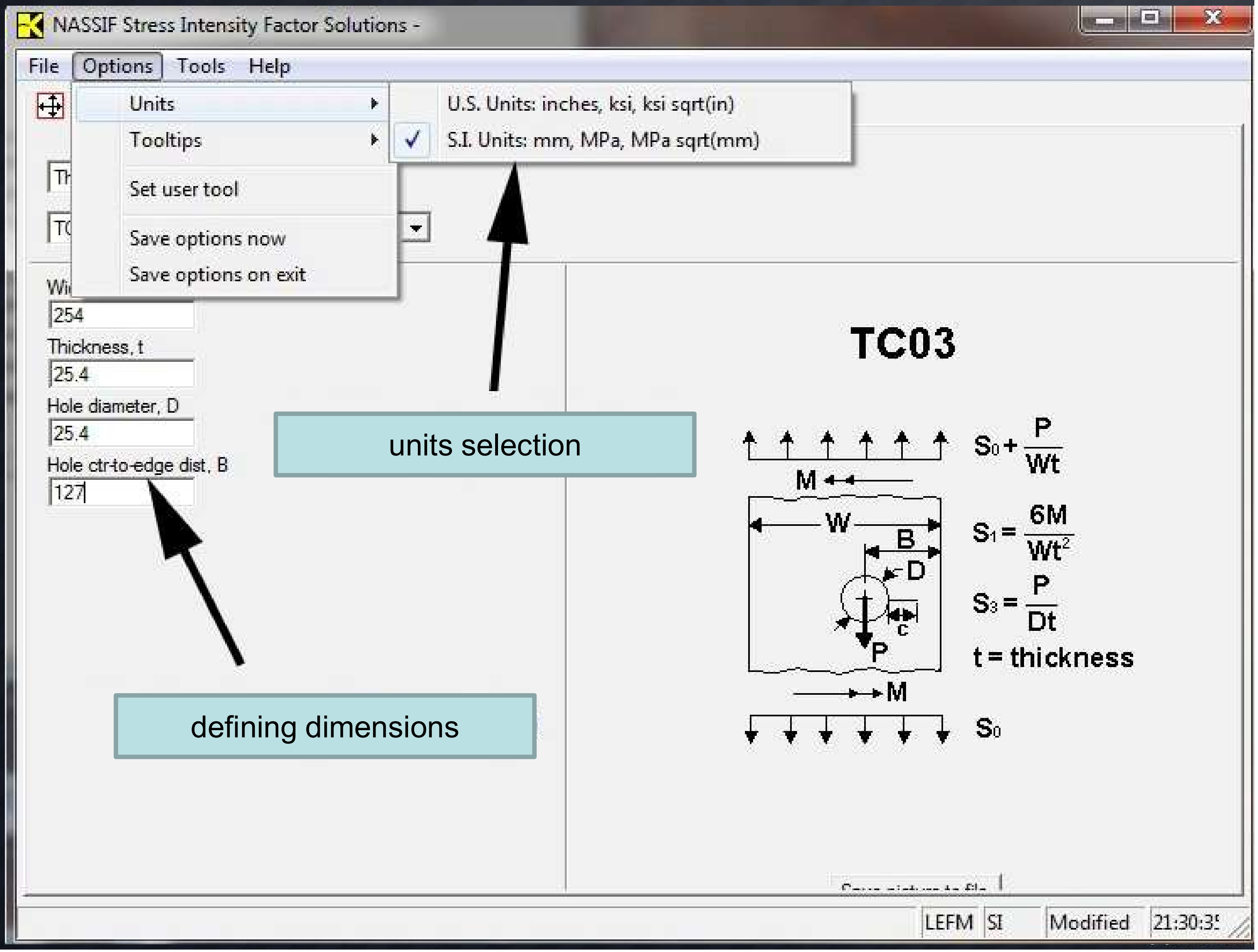
Source: Theo Fett, Stress Intensity Factors – T-Stresses – Weight Functions (2008)

Software for numerical methods

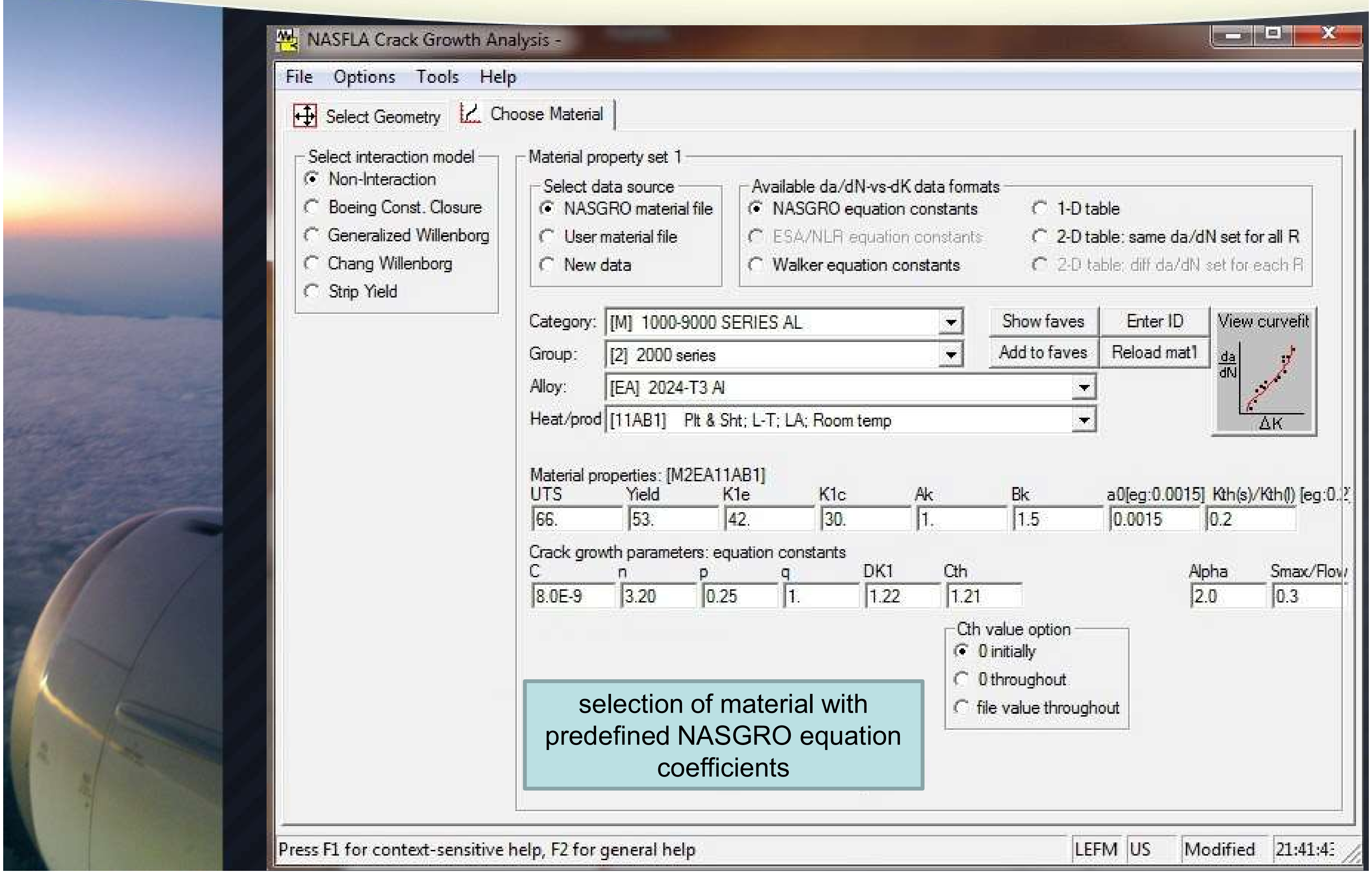
- NASGRO v4 (boundary element method)
- FRANC2D (finite element method)
- FRANC3D (finite element method)
- Ansys (finite element method)
- MSC Fatigue (finite element method)
- Abaqus (extended FEM)
- Code Aster (extended FEM)
- Zebulon (extended FEM).

https://www.swri.org/consortia/nasgro

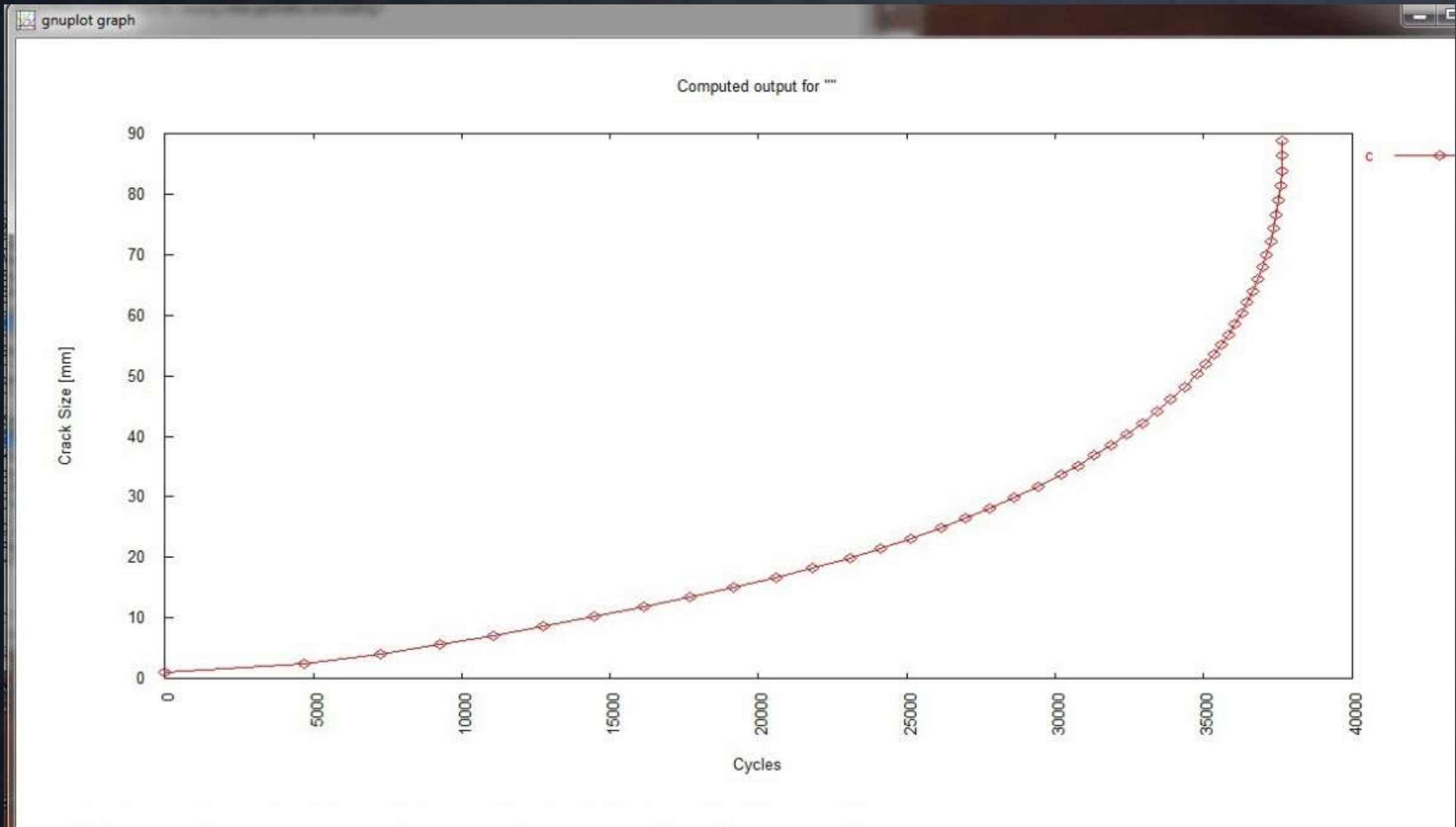
NASGRO v4 software (2D models)



NASGRO v4 material selection

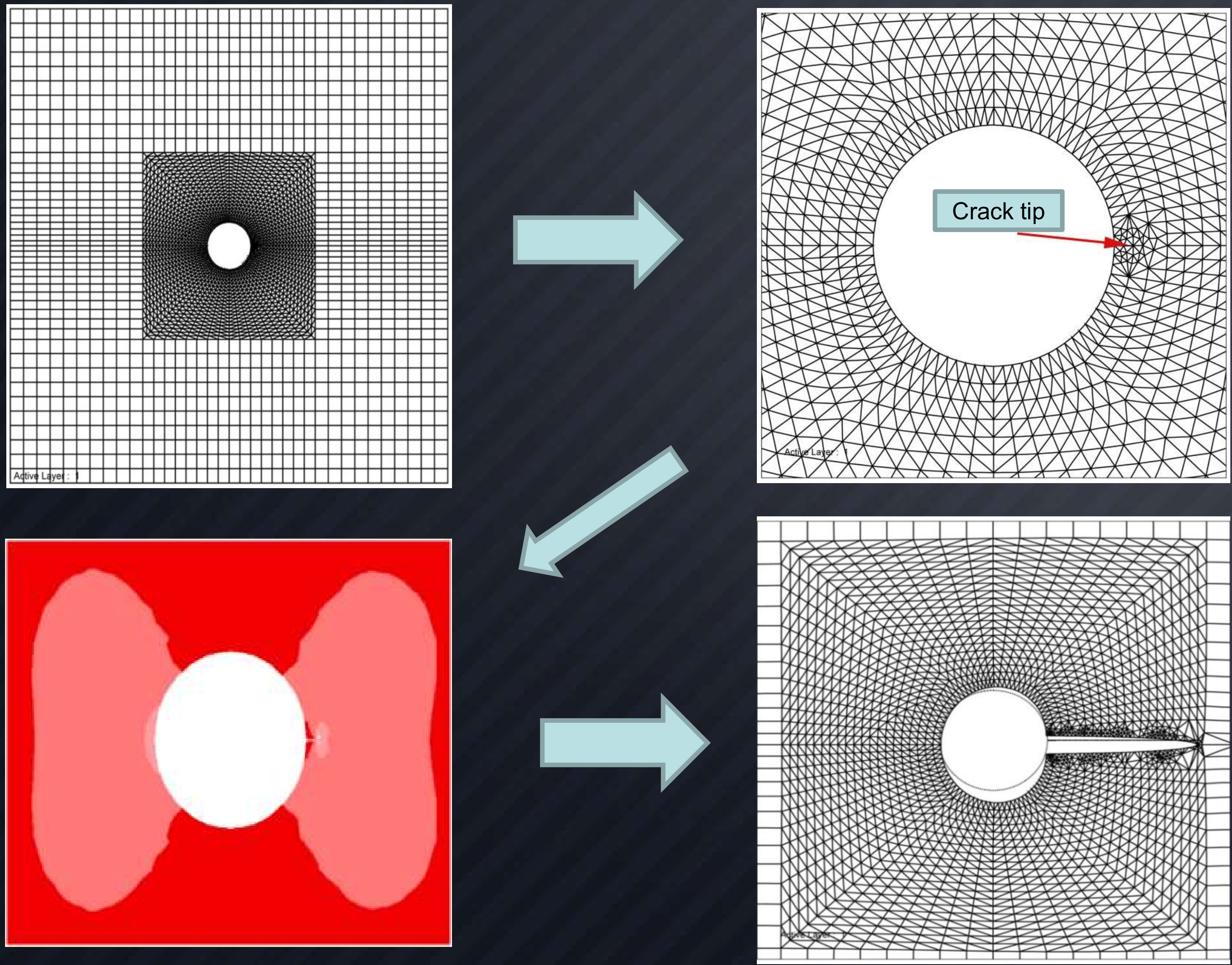


NASGRO v4 output files

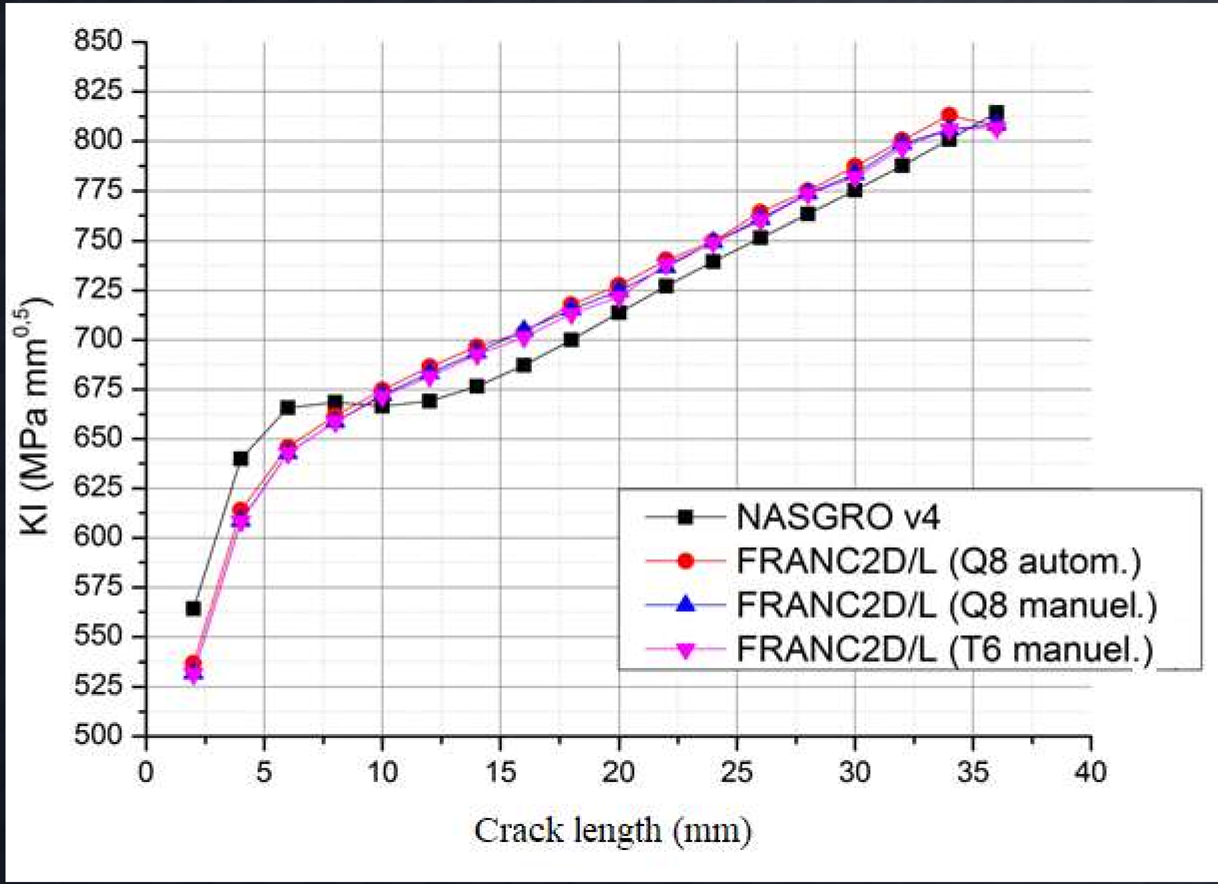


https://cfg.cornell.edu/software/

FRANC2D/L (FME model)

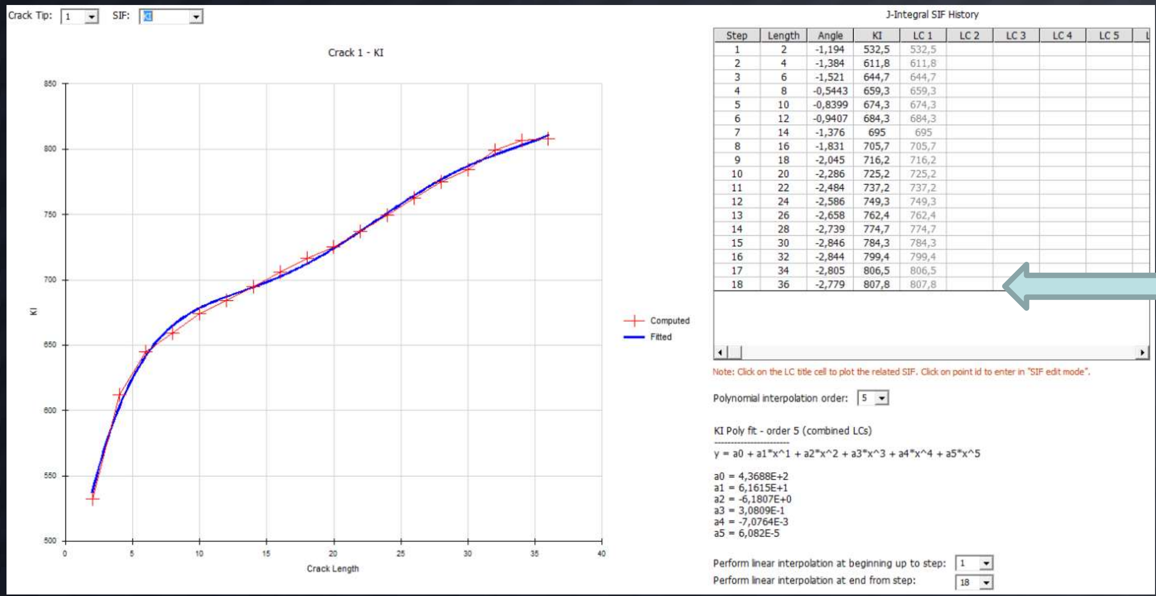


NASGRO i FRANC2D/L comparison

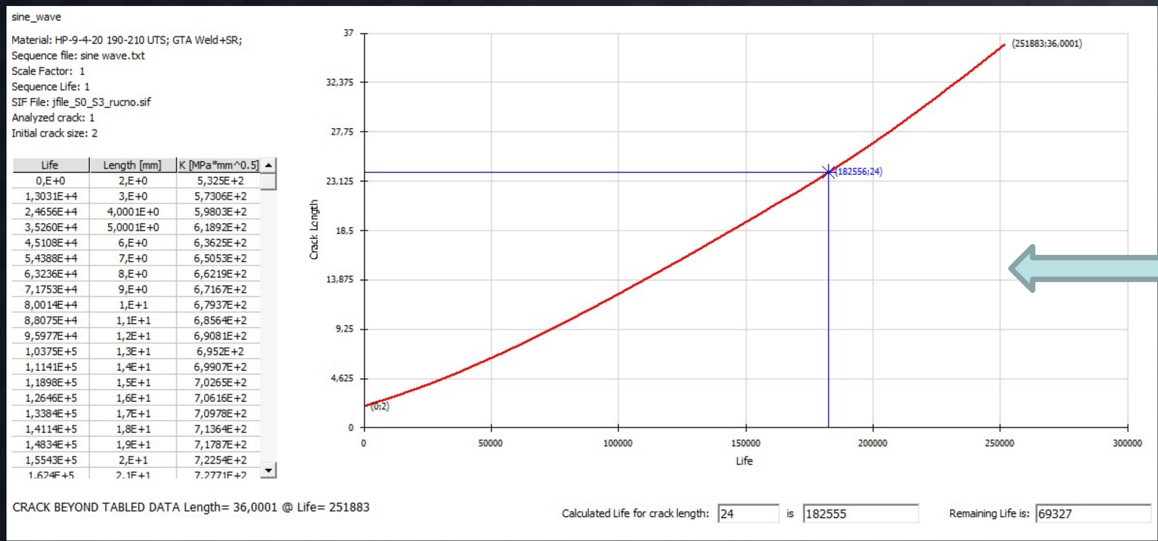


K_I values obtained in NASGRO i FRANC2D/L (Q8 i T6 elements)

Calculation of fatigue life

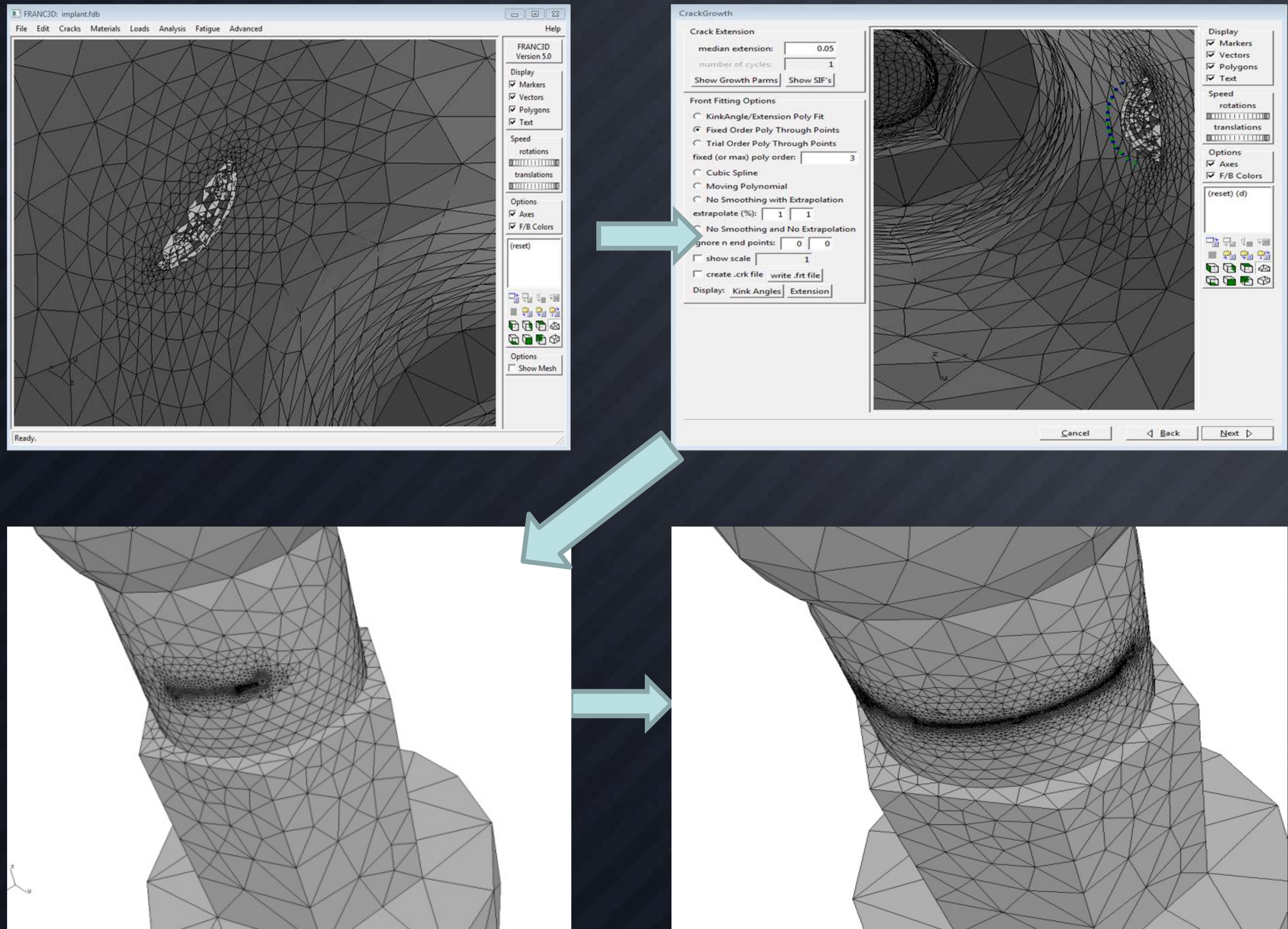


Defining polynomial equation for K_I change with crack length

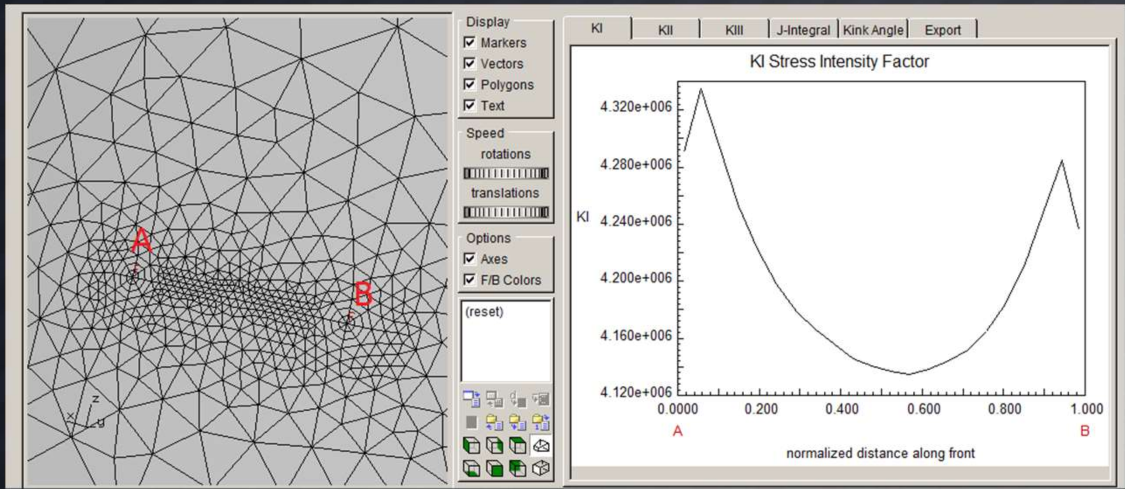


Integration of NASGRO equation (crack length as a function of number of cycles) https://franc3d.in/

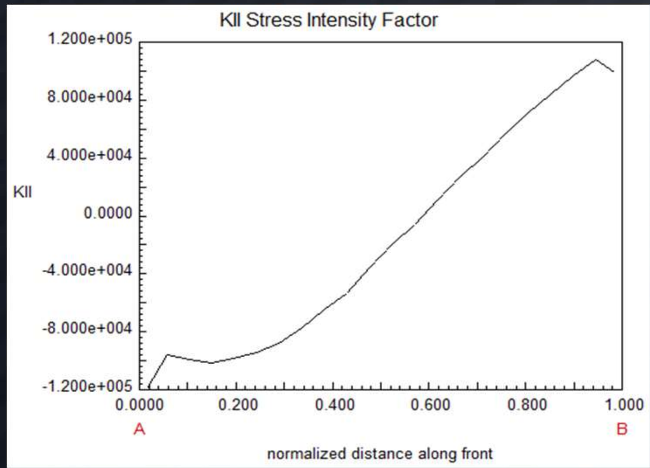
FRANC3D (FME model)



K_I and K_{II} calculated in FRANC3D



K_I values calculated along the crack front



K_{II} values calculated along the crack front

Equivalent SIFs and kink angle

- For automated crack growth the following equations for *equivalent SIF are used*:

$$K_{eq} = \sqrt[4]{K_I^4 + 8K_{II}^4}$$

$$K_{eq} = \frac{1}{B}\sqrt{(k_1)^2 + \left(\frac{k_2}{s}\right)^2 + A(k_H)^2}$$

$$K_{eq} = \sqrt{K_I^2 + K_{II}^2}$$

- Kink angle formula:

$$\theta = -arccos\left[\frac{2K_{II}^2 + K_I\sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2}\right]$$

https://www.ansys.com/

Ansys SMART technology (FEM)

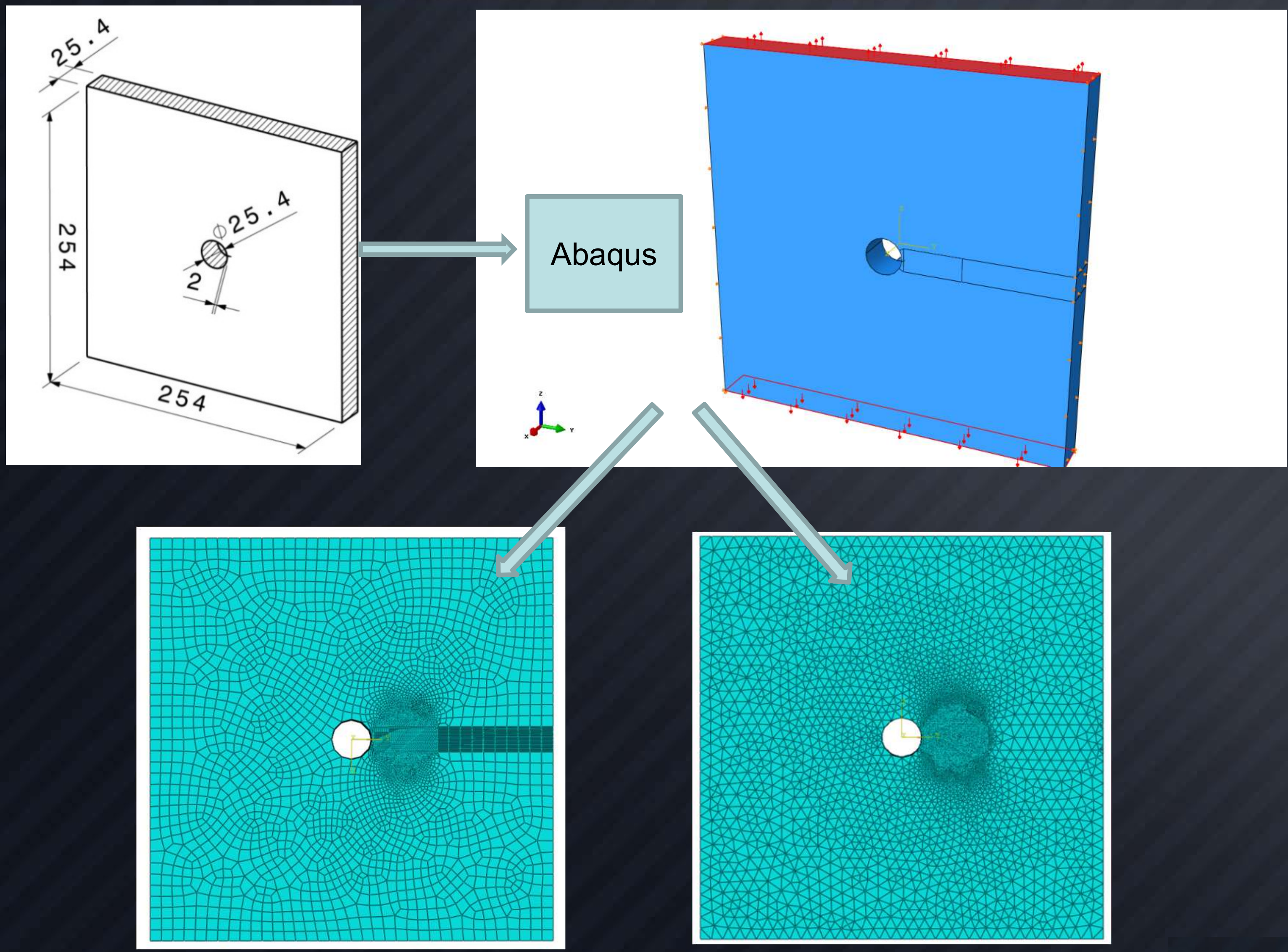
- SMART: Separating, Morphing, Adaptive and Re-meshing Technology
- Calculates Mode I, II, III Stress Intensity Factors (SIFs)
- Supports static crack propagation based on failure criteria using SIFs or J-Integral
- Supports fatigue crack propagation based on Paris' law
- Supports crack propagation of internally generated crack meshes including semi-elliptical and arbitrary cracks
- Supports crack propagation of pre-meshed cracks
- Limited to isotropic linear elastic analyses (no plasticity, no nonlinear geometry effects, no load-compression effects, no crack-tip-closure effects)
- Assemblies are supported, but only *MPC* (multi point constraint) formulation can be used (no frictional or frictionless contact)
- Supports multiple cracks in the model
- Thermal loads and imported loads (pressure) can be used.

Extended FEM (XFEM)

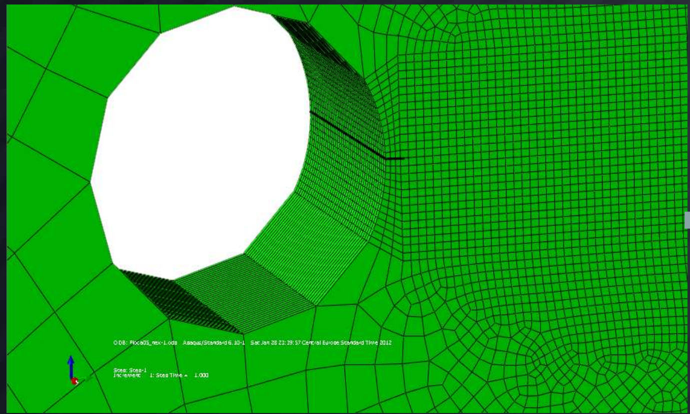
- Enables the modeling of a discontinuous field independently of the generated finite element mesh.
- XFEM does not require mapping between the mesh and geometry of discontinuity .
- It is possible to use an arbitrary crack shape, and the fatigue crack growth simulation can be performed without generating new nodes around the tip as the crack grows.

https://edu.3ds.com/en/software/abaqus-student-edition

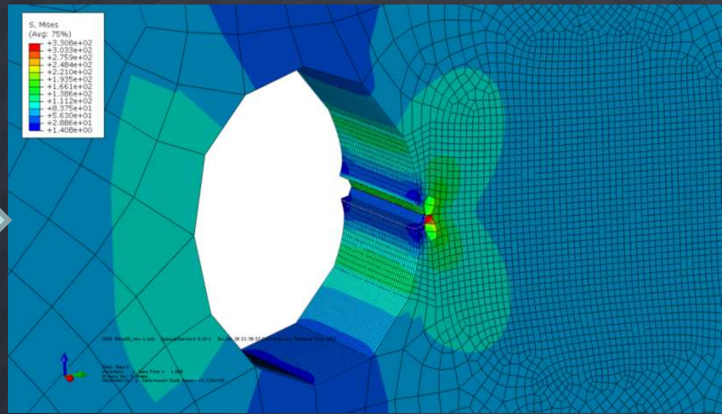
XFEM verification – Ex. 1 (TC03)



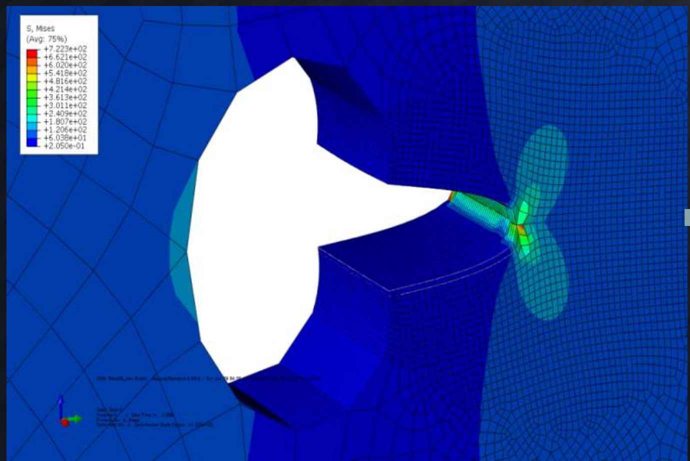
XFEM verification – Ex. 1 (TC03)



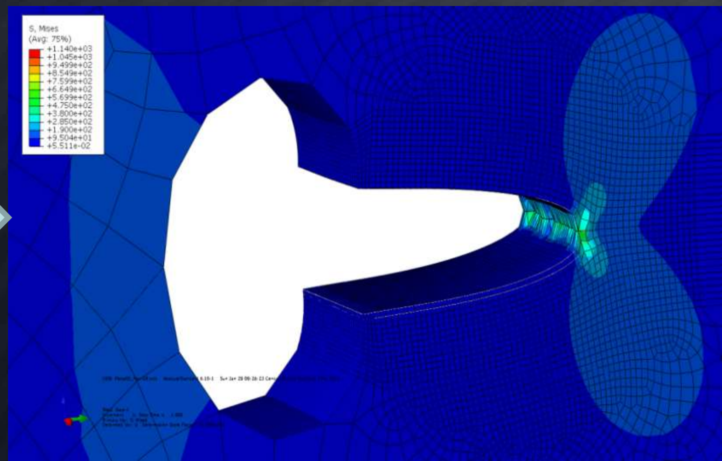
Initial crack



Crack „opening“

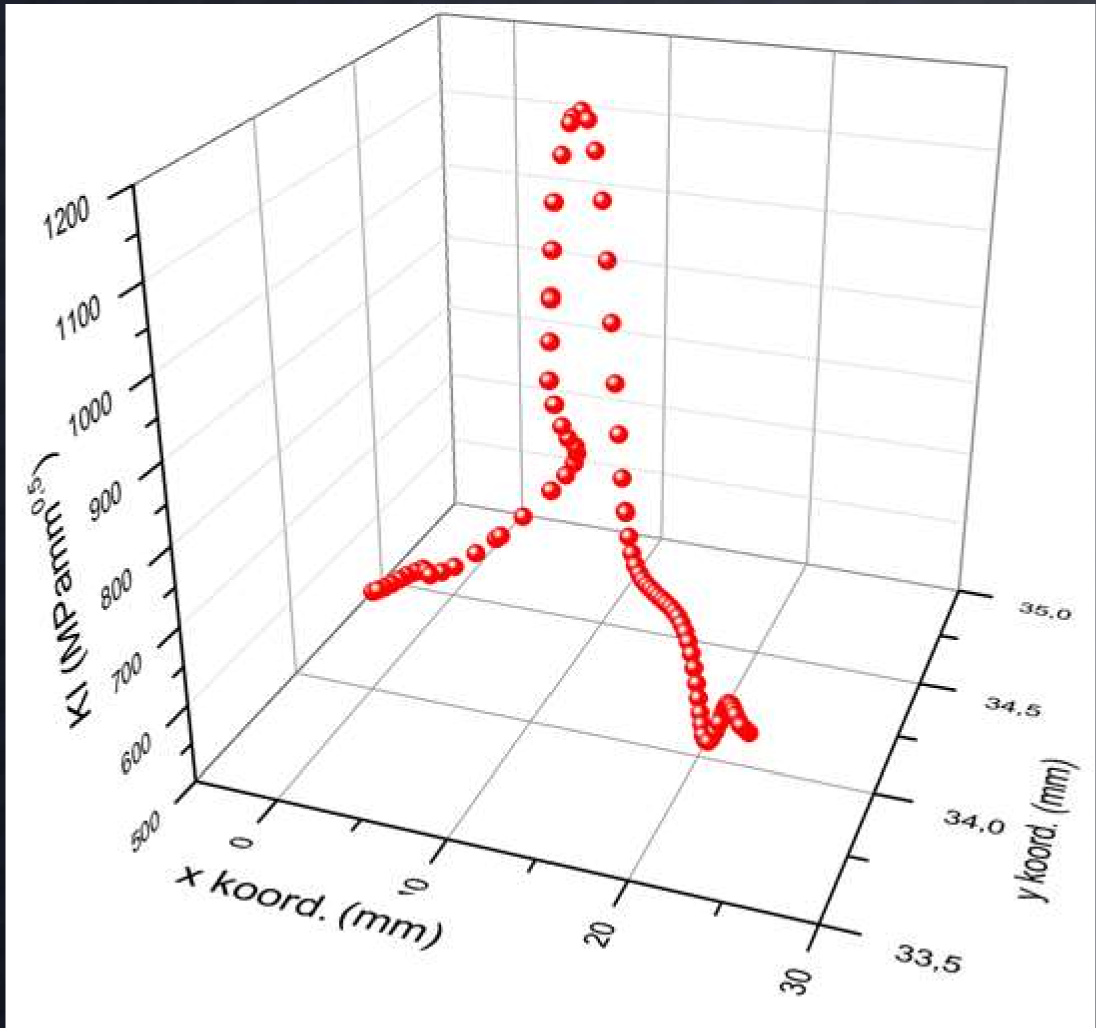


After 9th step of growth



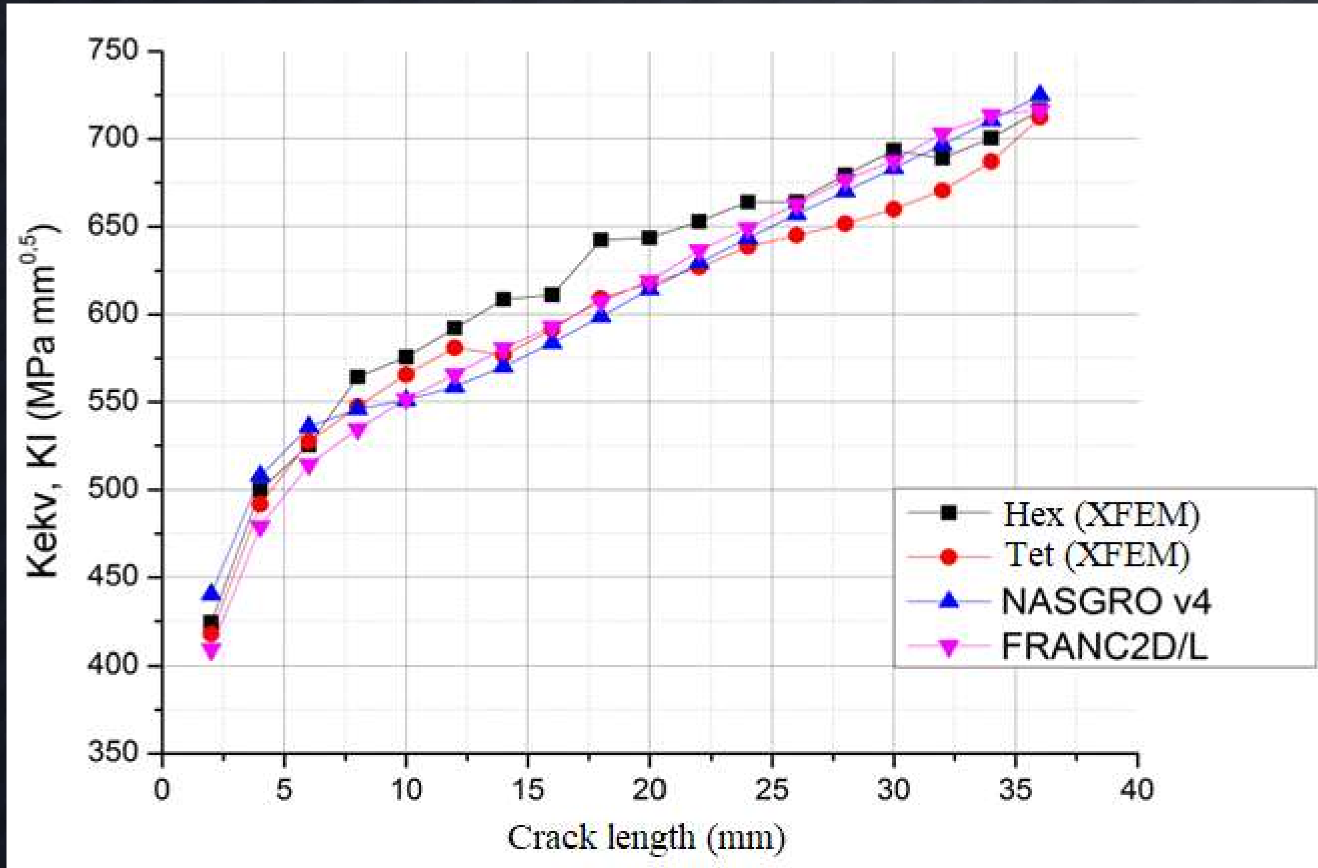
After 17th step

Ex. 1 – K_I values along crack front



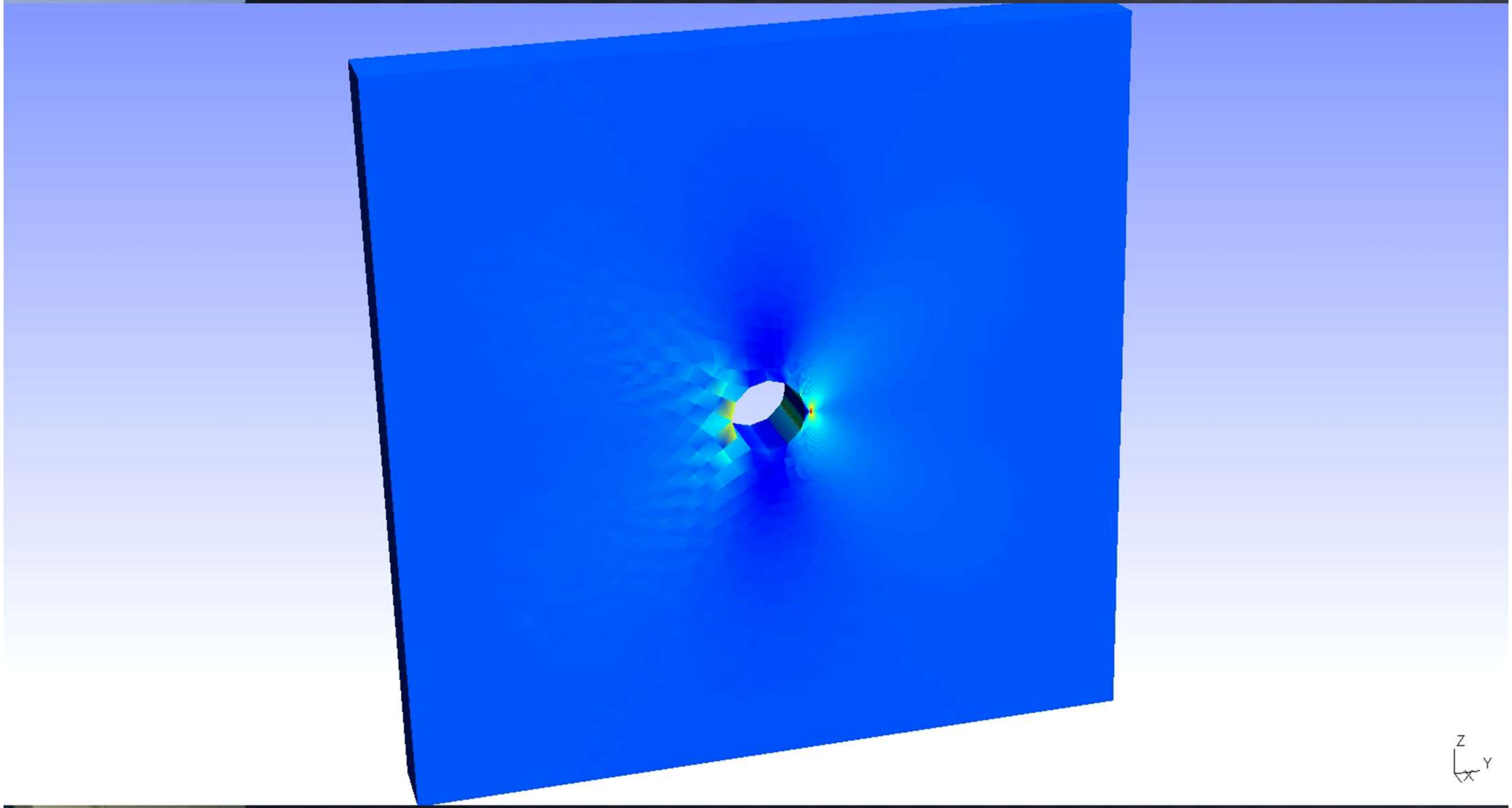
K_I values along crack front after 12th step of growth

Ex. 1 : K_I and K_{eqv} values comparison

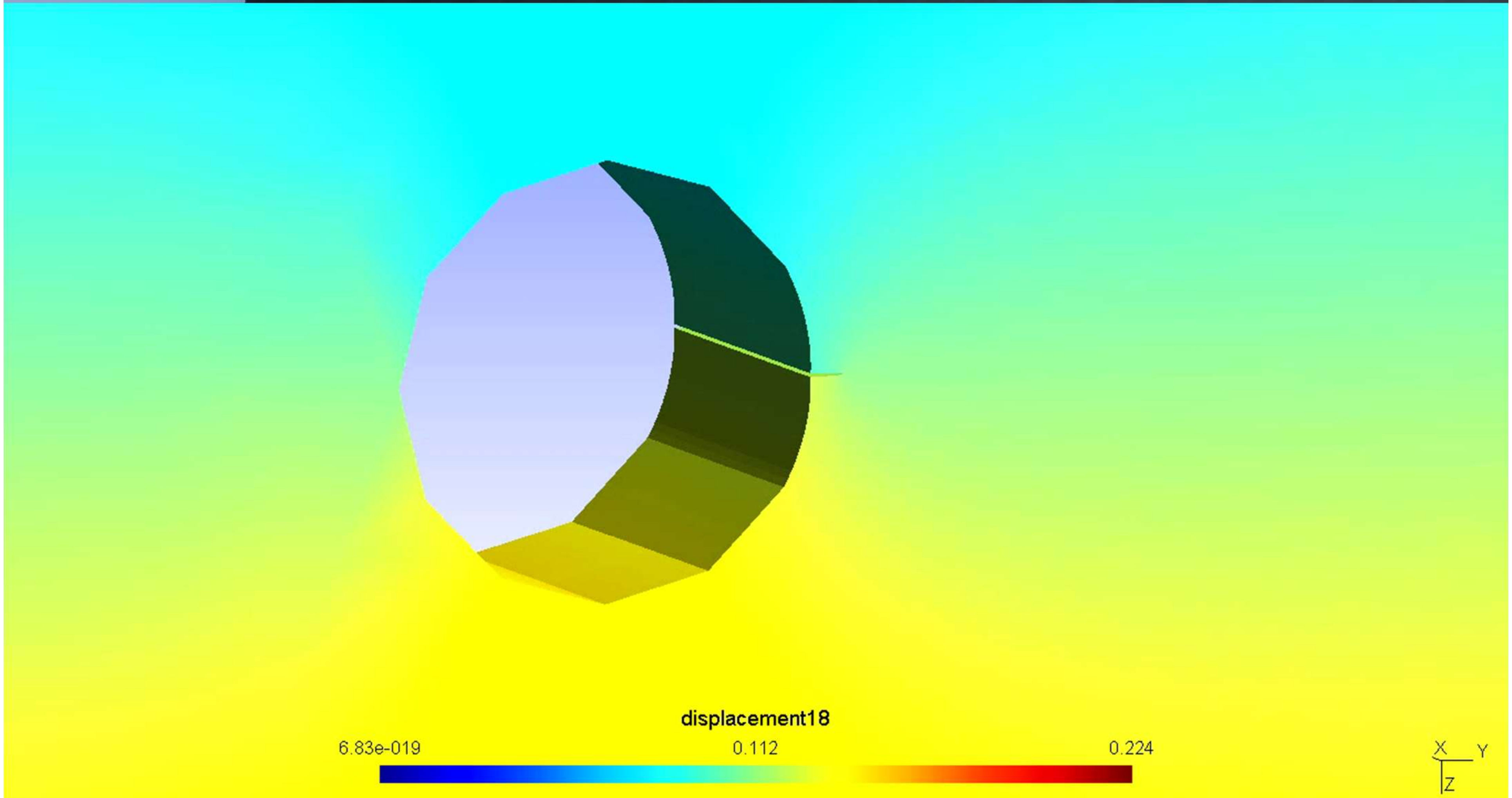


Mean K_{eqv} values (XFEM - Abaqus) vs. K_I (FRANC2D/L and NASGRO)

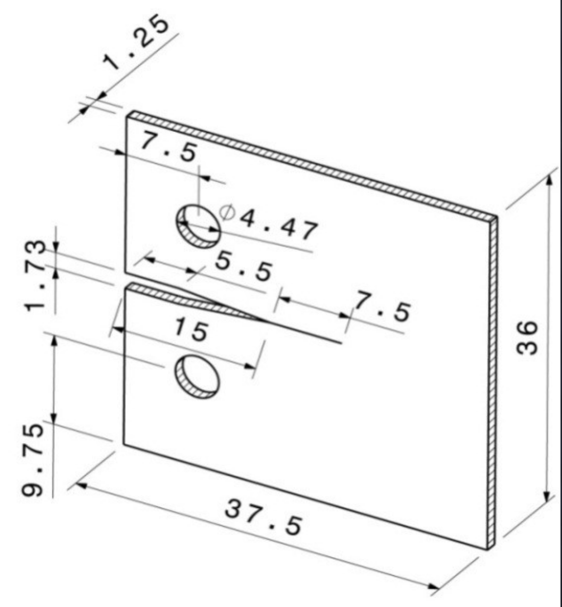
Ex. 1 – Stress change with growth

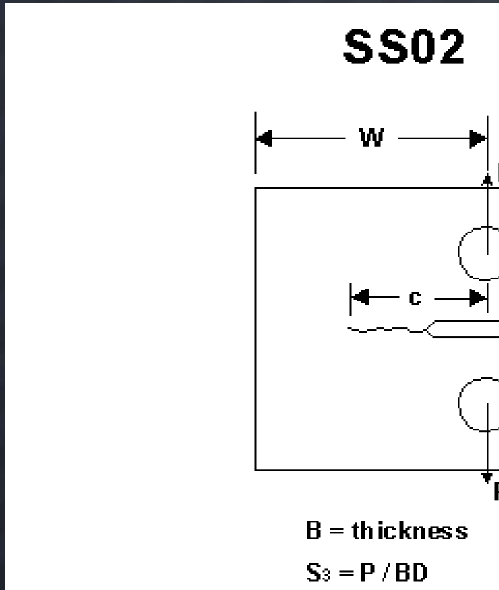


Ex. 1 – Displacement field



Ex. 2 – CT specimen SIF calculation





S_3=35,777MPa

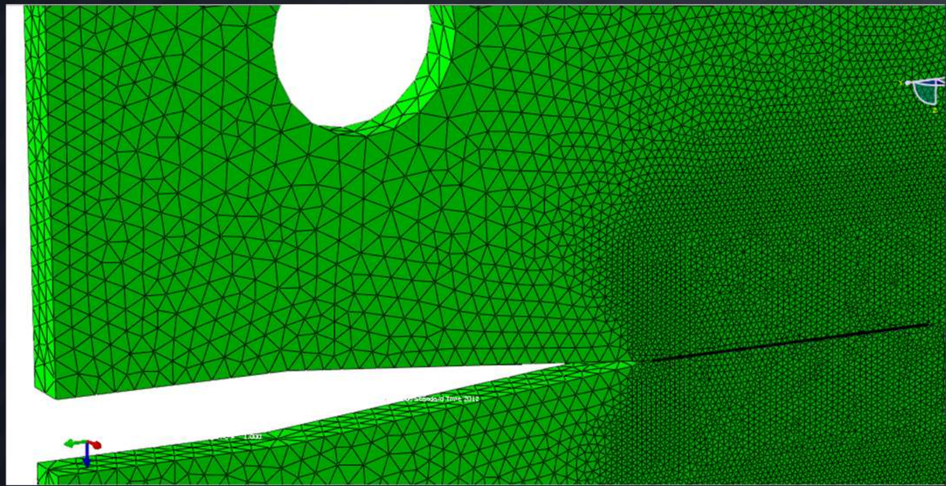
Dimensions and load of CT specimen

$$K_I^{(theor)} = \frac{P \cdot Y(c/w)}{B \cdot w^{1/2}} \qquad 0{,}3 \leq \left(\frac{c}{w}\right) \leq 0{,}7$$

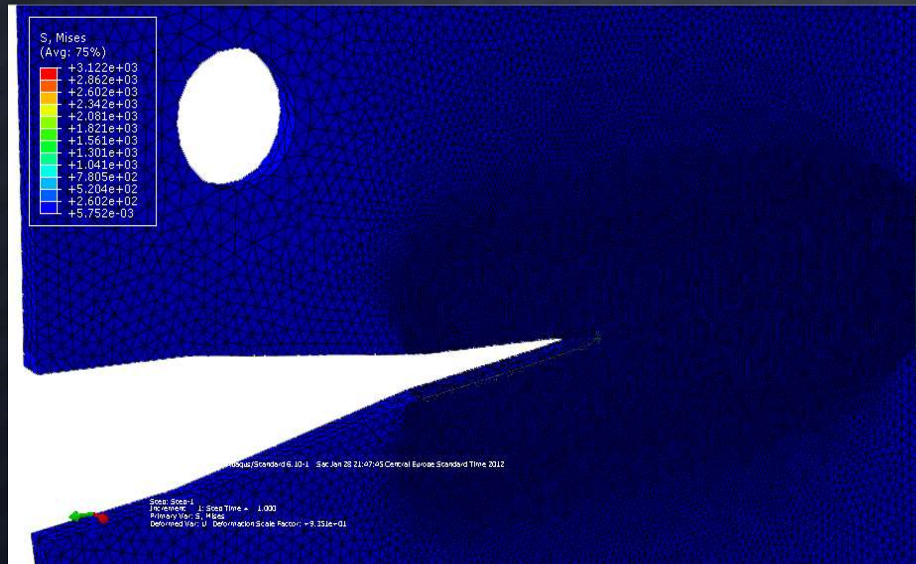
$$Y(c/w) = 39{,}7\left(\frac{c}{w}\right)^{\frac{1}{2}} - 294\left(\frac{c}{w}\right)^{\frac{3}{2}} + 1118\left(\frac{c}{w}\right)^{\frac{5}{2}} - 1842\left(\frac{c}{w}\right)^{\frac{7}{2}} + 1159\left(\frac{c}{w}\right)^{\frac{9}{2}}$$

$$K_I^{(theor)} = \frac{200N \cdot 10.17348}{1.25mm \cdot (30mm)^{1/2}} = 297{,}187MPamm^{0,5}$$

Ex. 2 – CT BEM and XFEM results

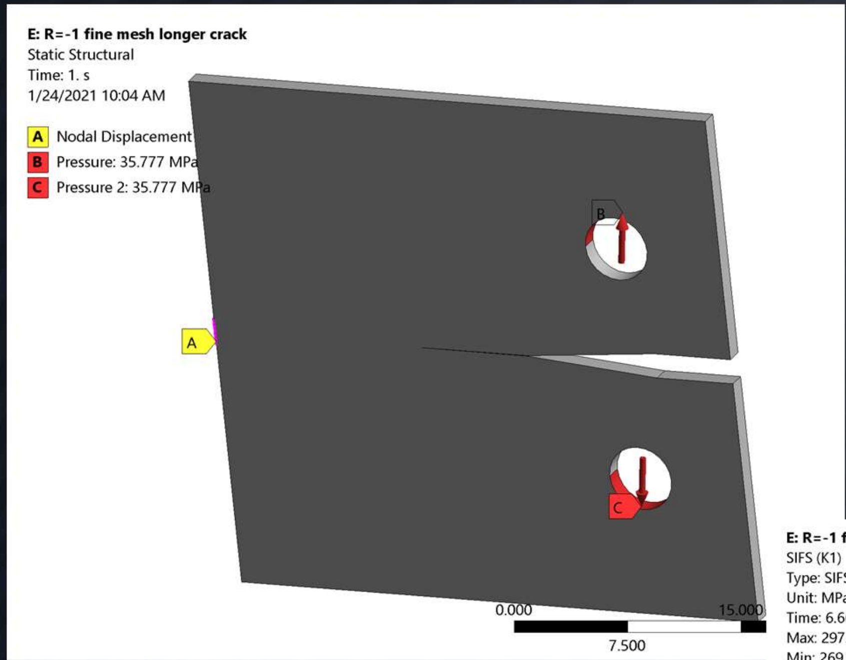


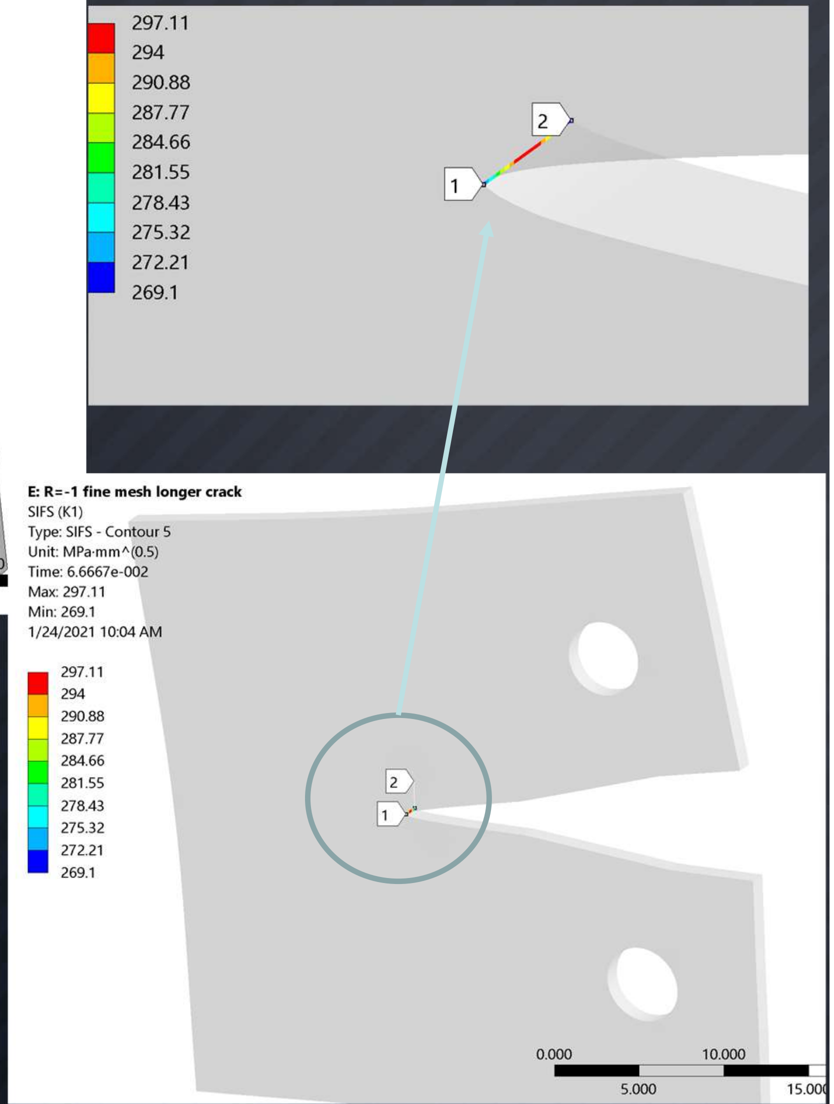
Abaqus result (XFEM):
K_I =292,50$MPamm^{0,5}$
(difference 1.57%)



NASGRO v4 result (BEM):
K_I = 282,024$MPamm^{0,5}$
(difference 5.1%)

Ex. 2 – CT specimen FEM result



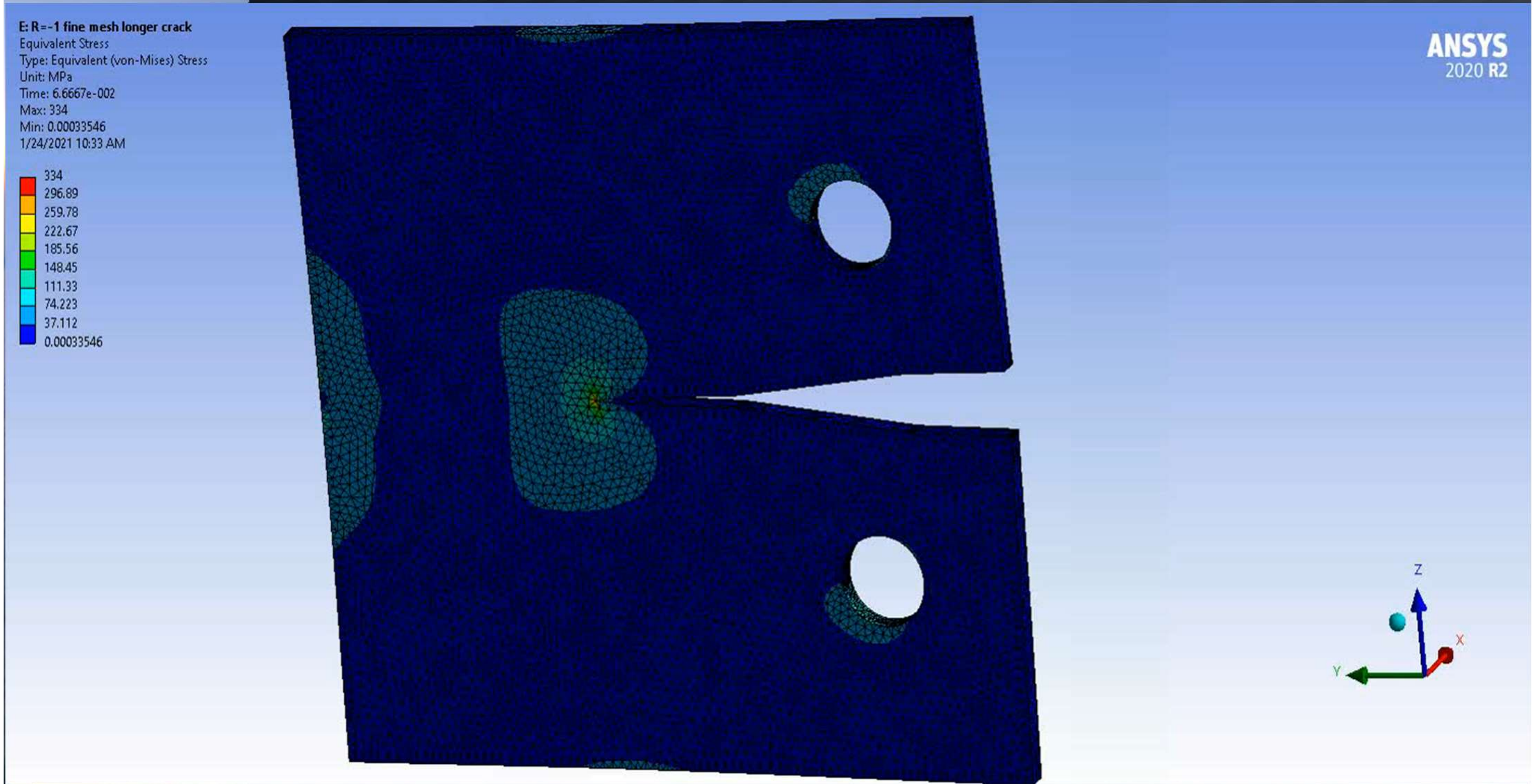


Ansys result (FEM):

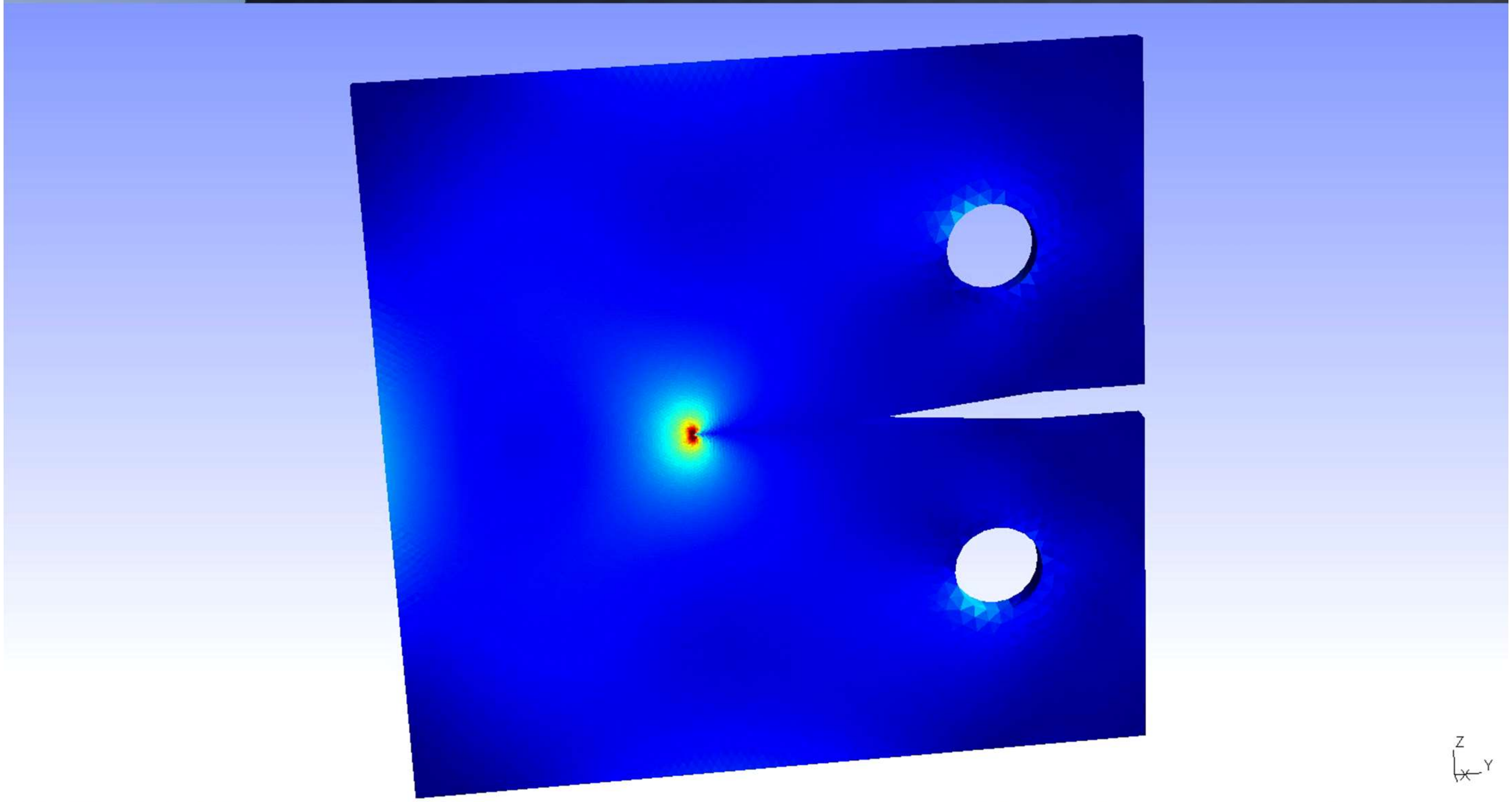
$K_I = 297{,}11 MPamm^{0,5}$

(difference 0.026%)

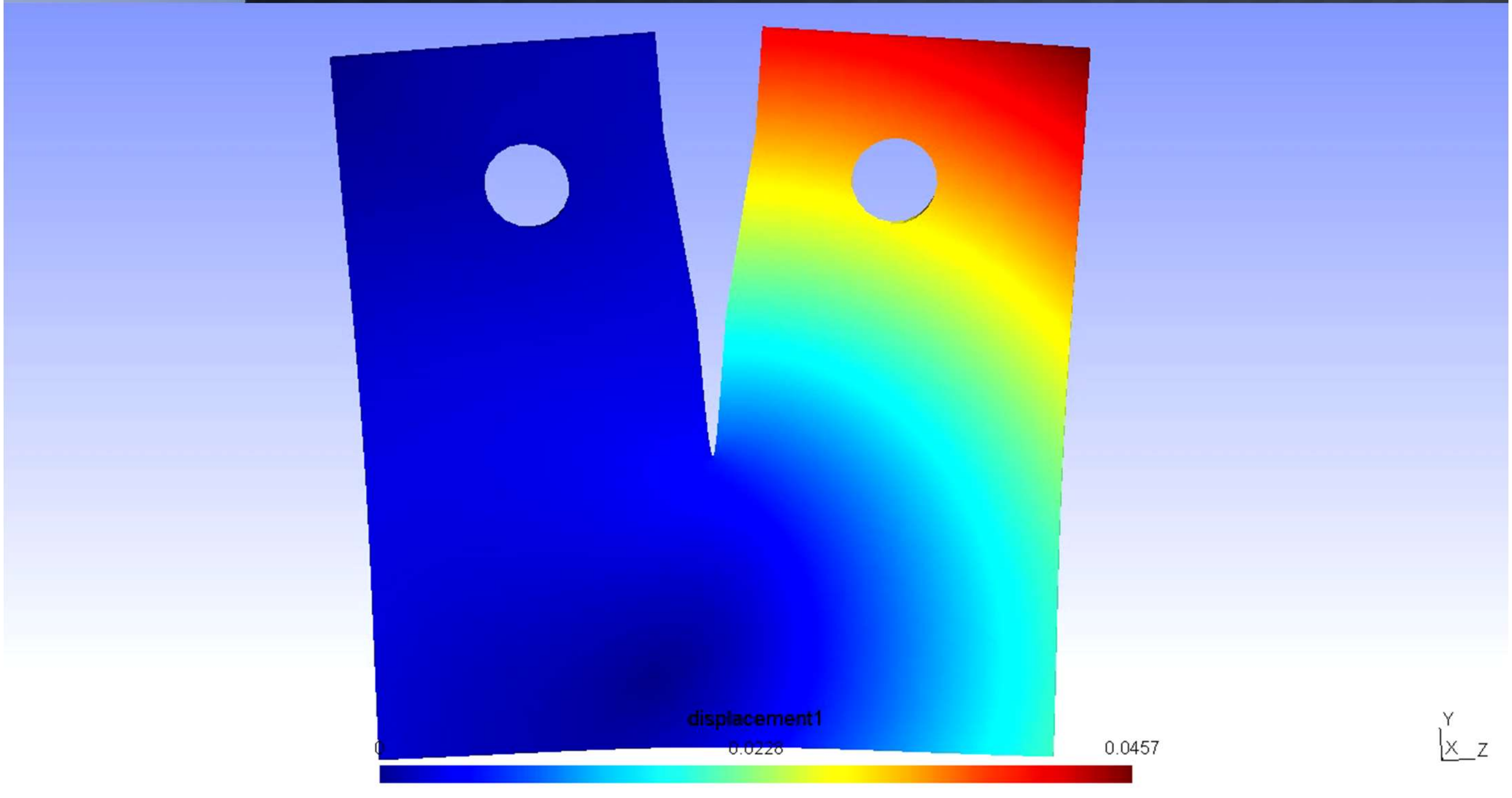
Ex. 2 – CT specimen stress (FEM)



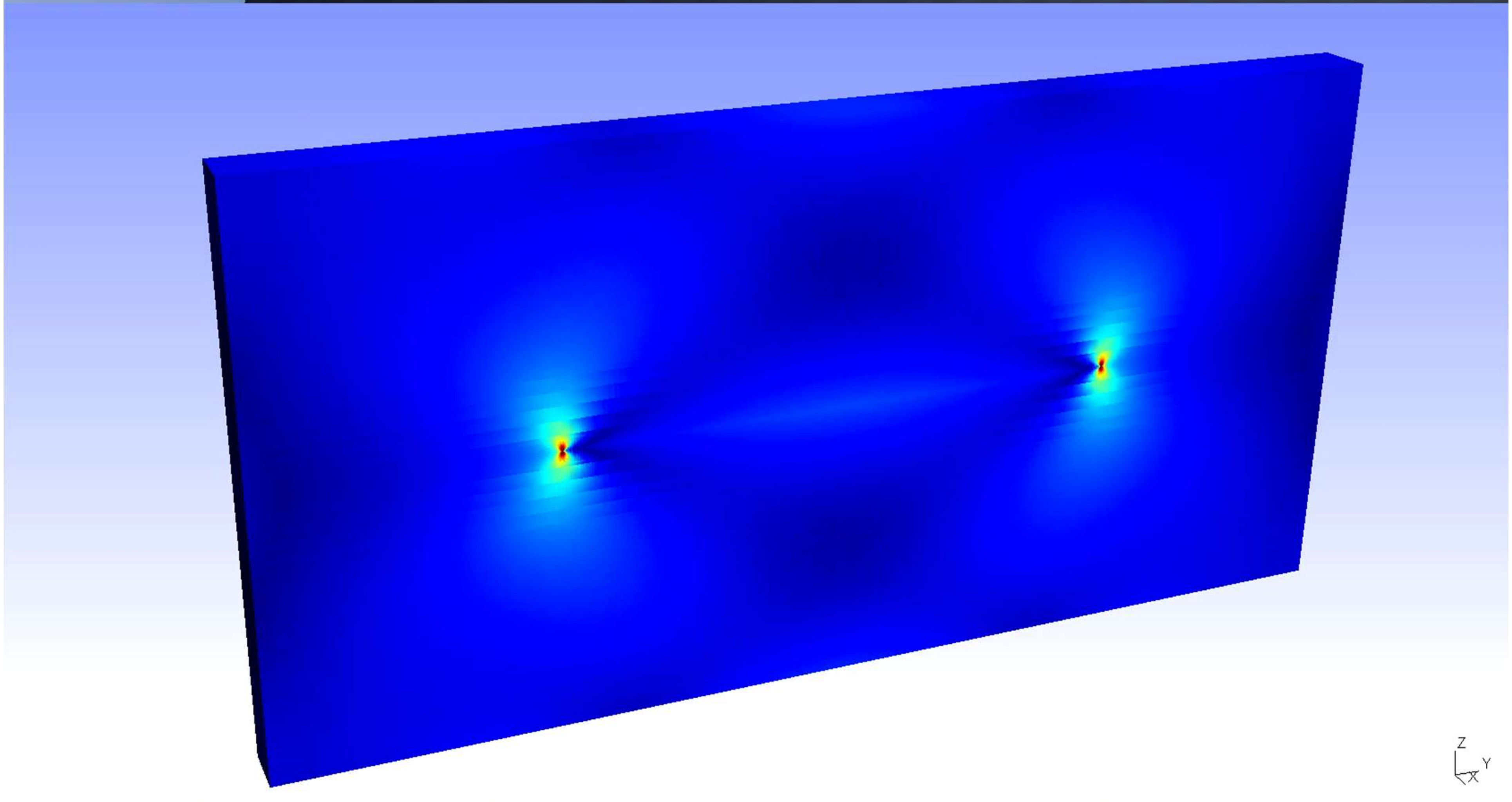
Ex. 2 – CT specimen stress (XFEM)



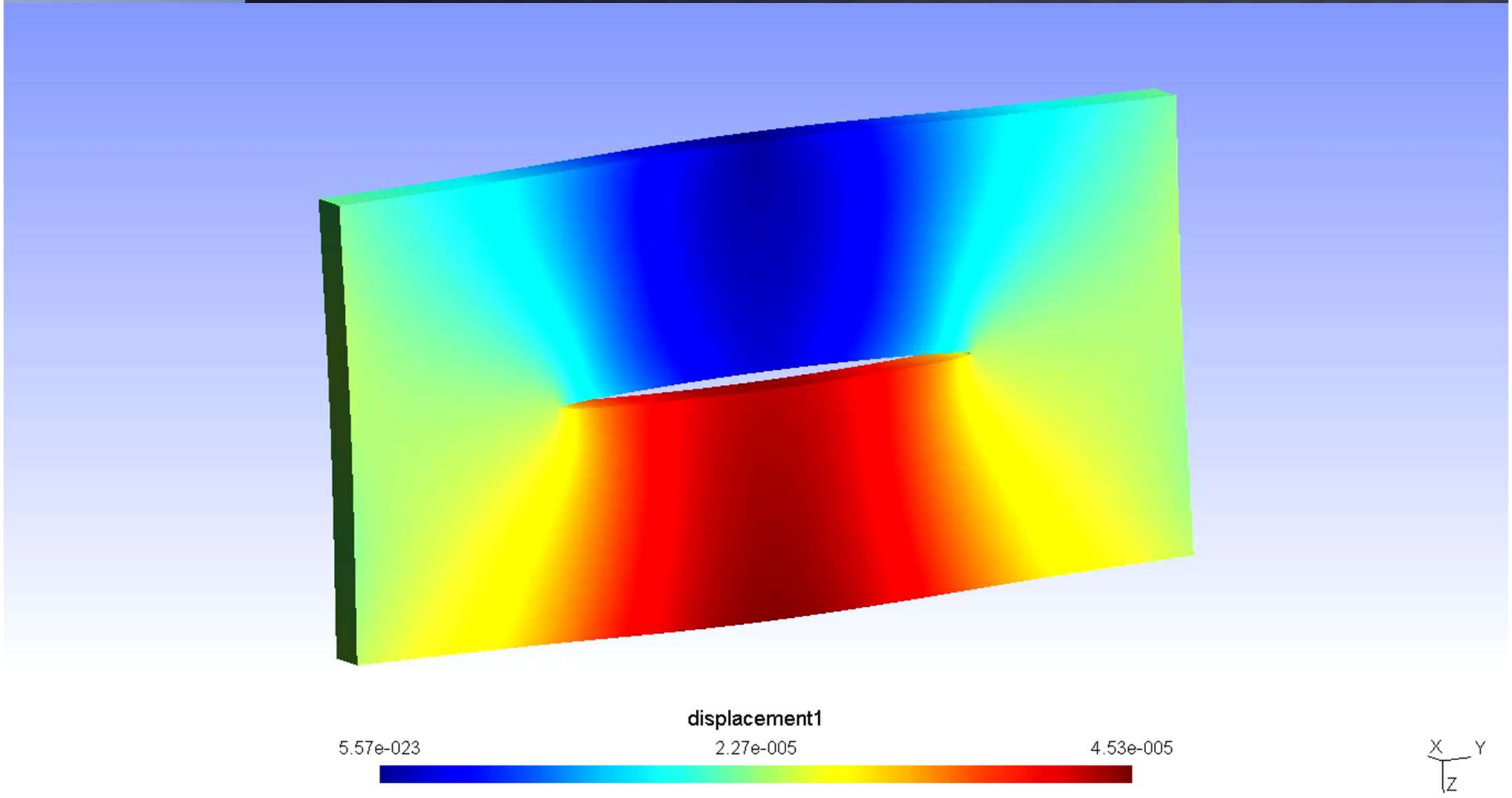
Ex. 2 – CT specimen (displacement)



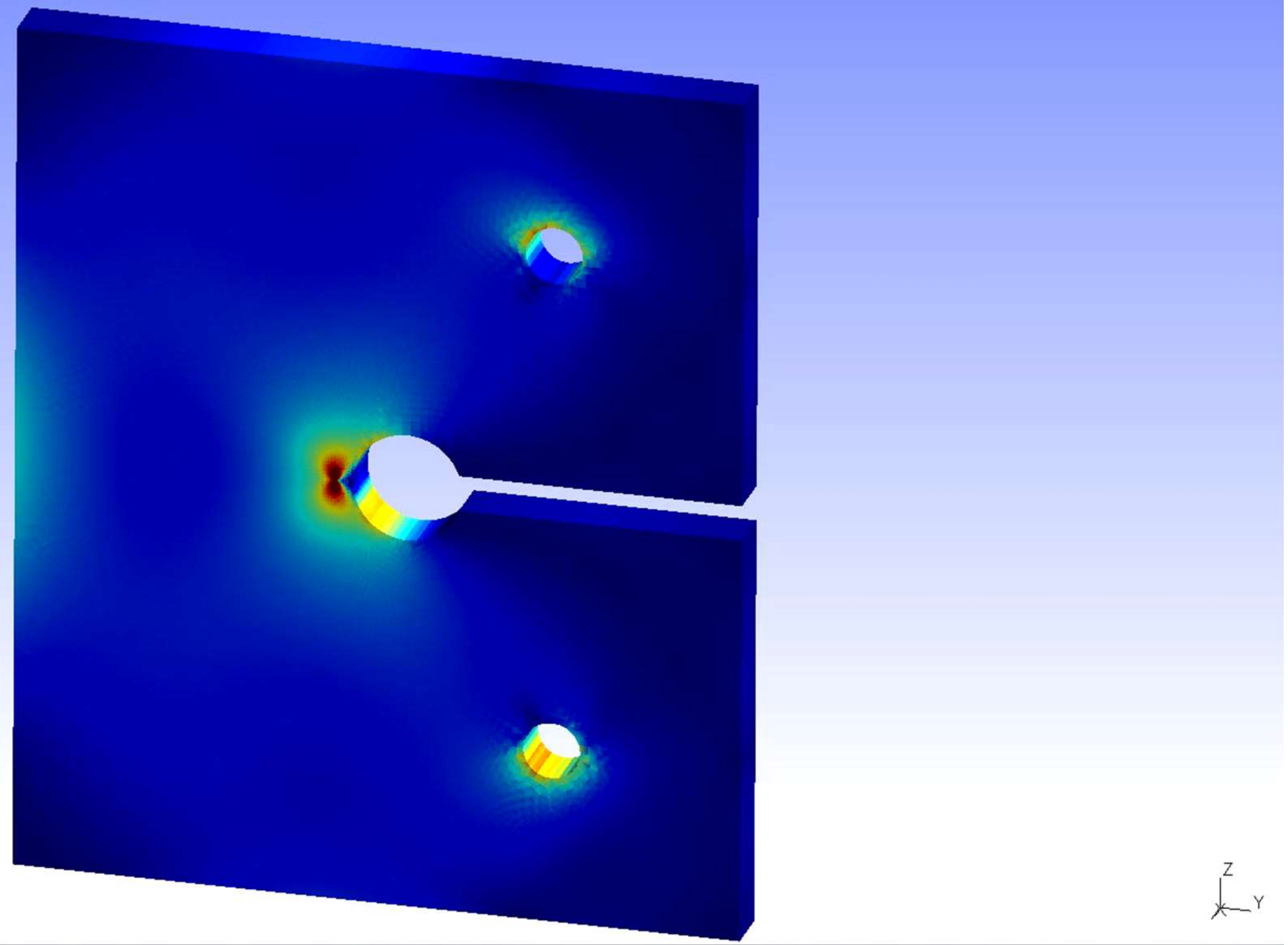
Ex. 3 – CCT specimen (stress field)



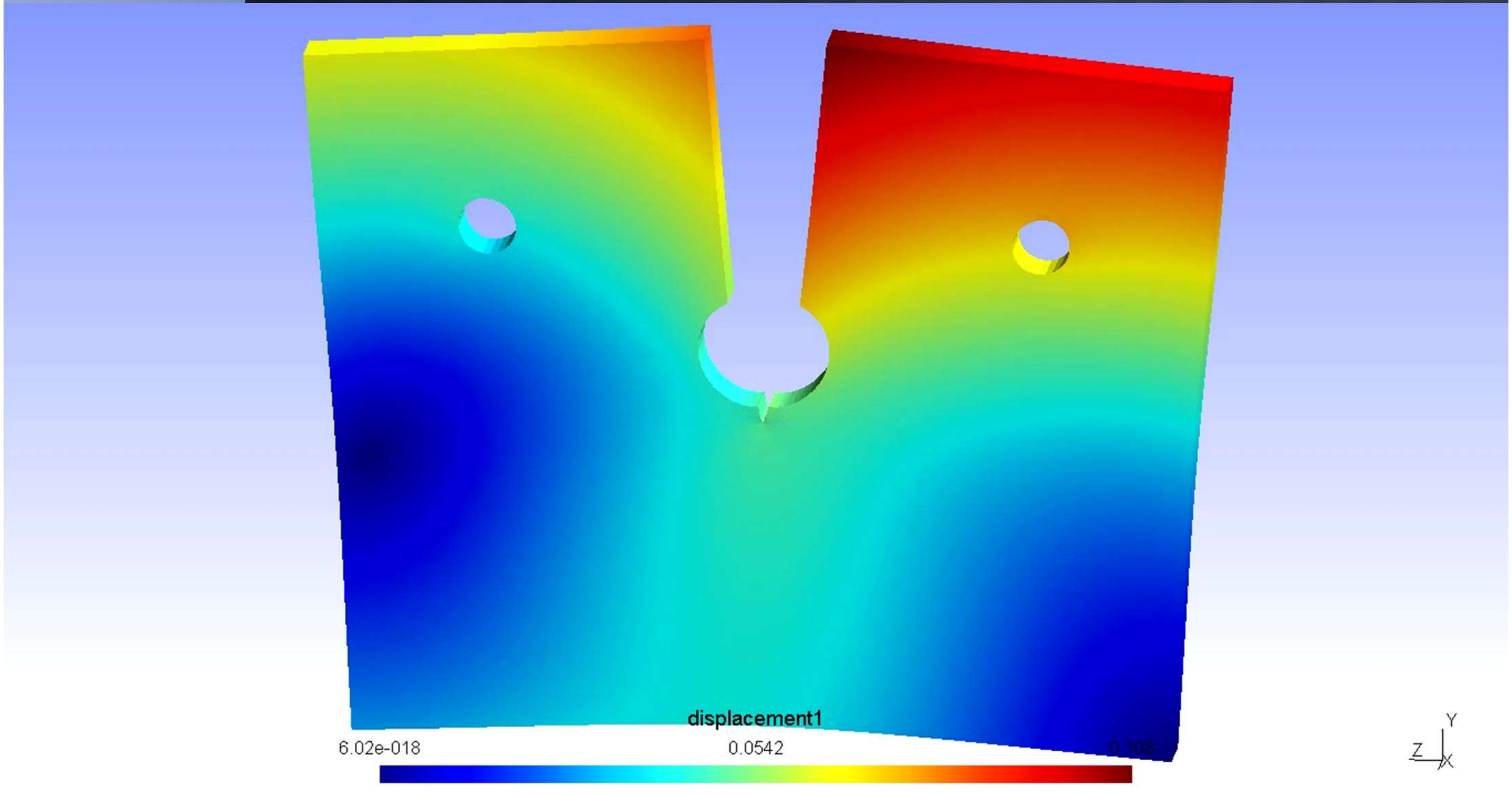
Ex. 3 – CCT specimen (displacement)



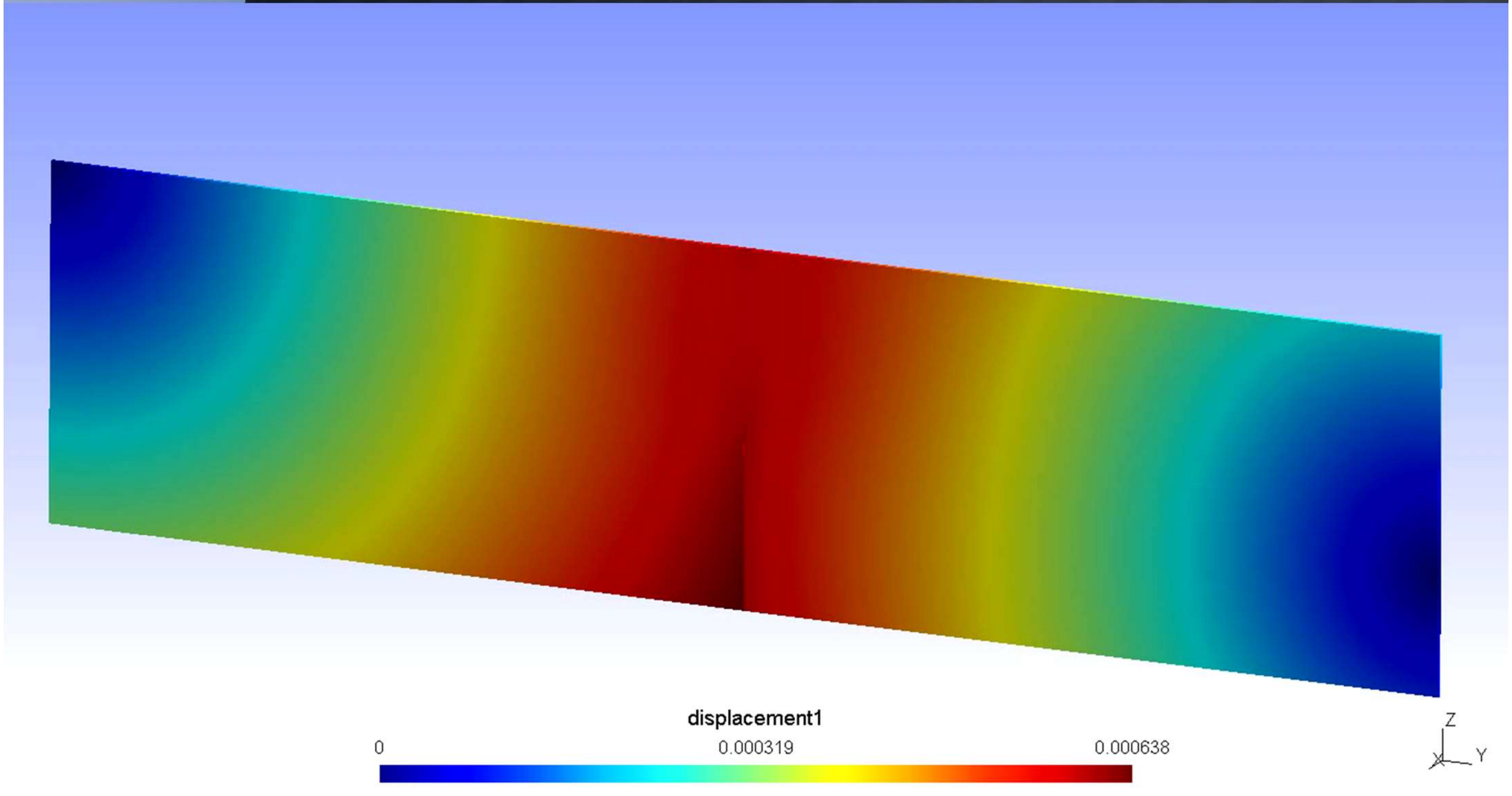
Ex. 4 – Non-standard specimen (stress field)



Ex. 4 – non-standard specimen (displacement)



Ex. 5 – Three-point flexural test

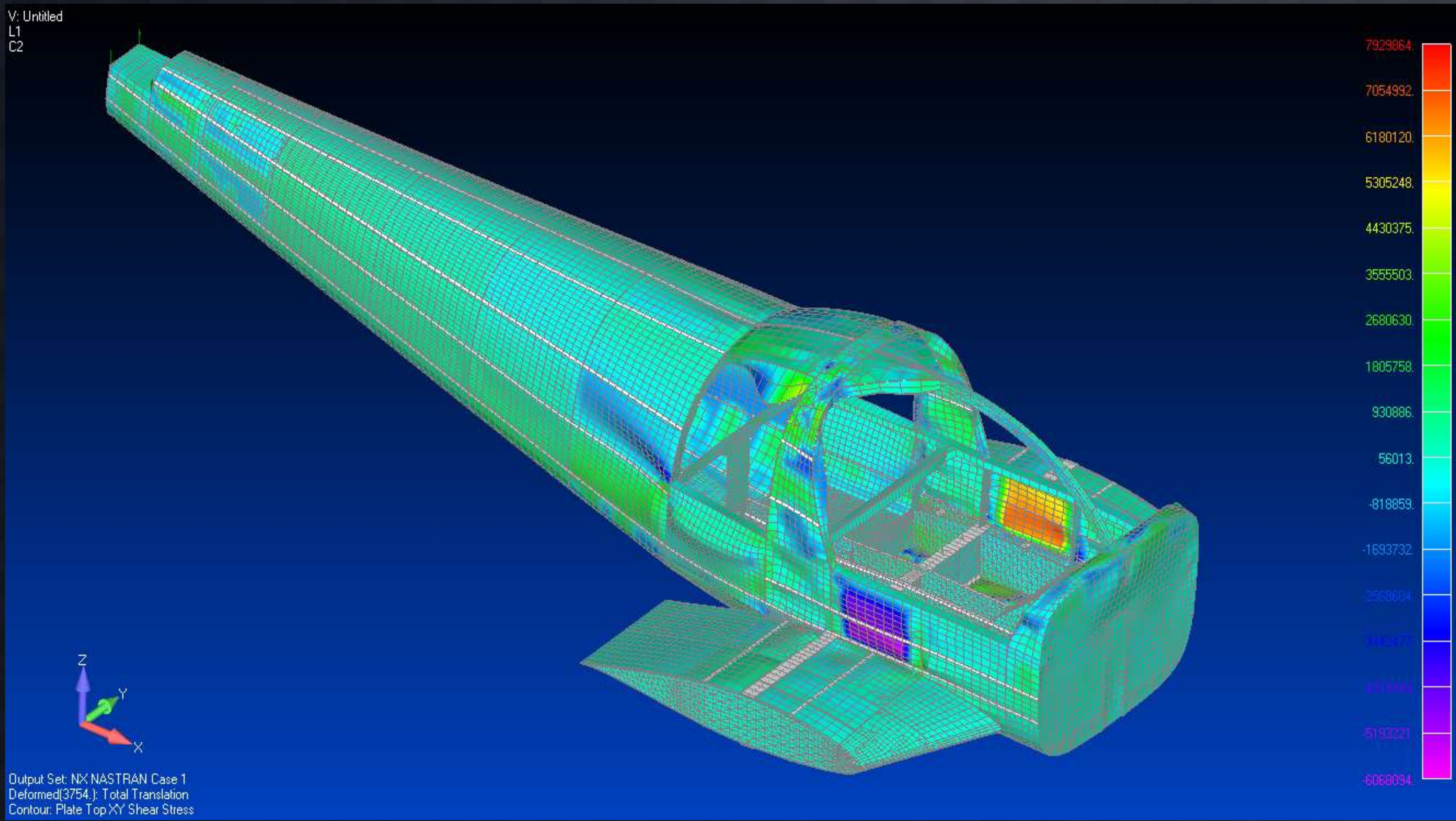


CASE STUDIES

- Damaged wing-fuselage attachment
- Crack growth in the wing spar
- Fatigue life assessment of damaged integral skin–stringer panel

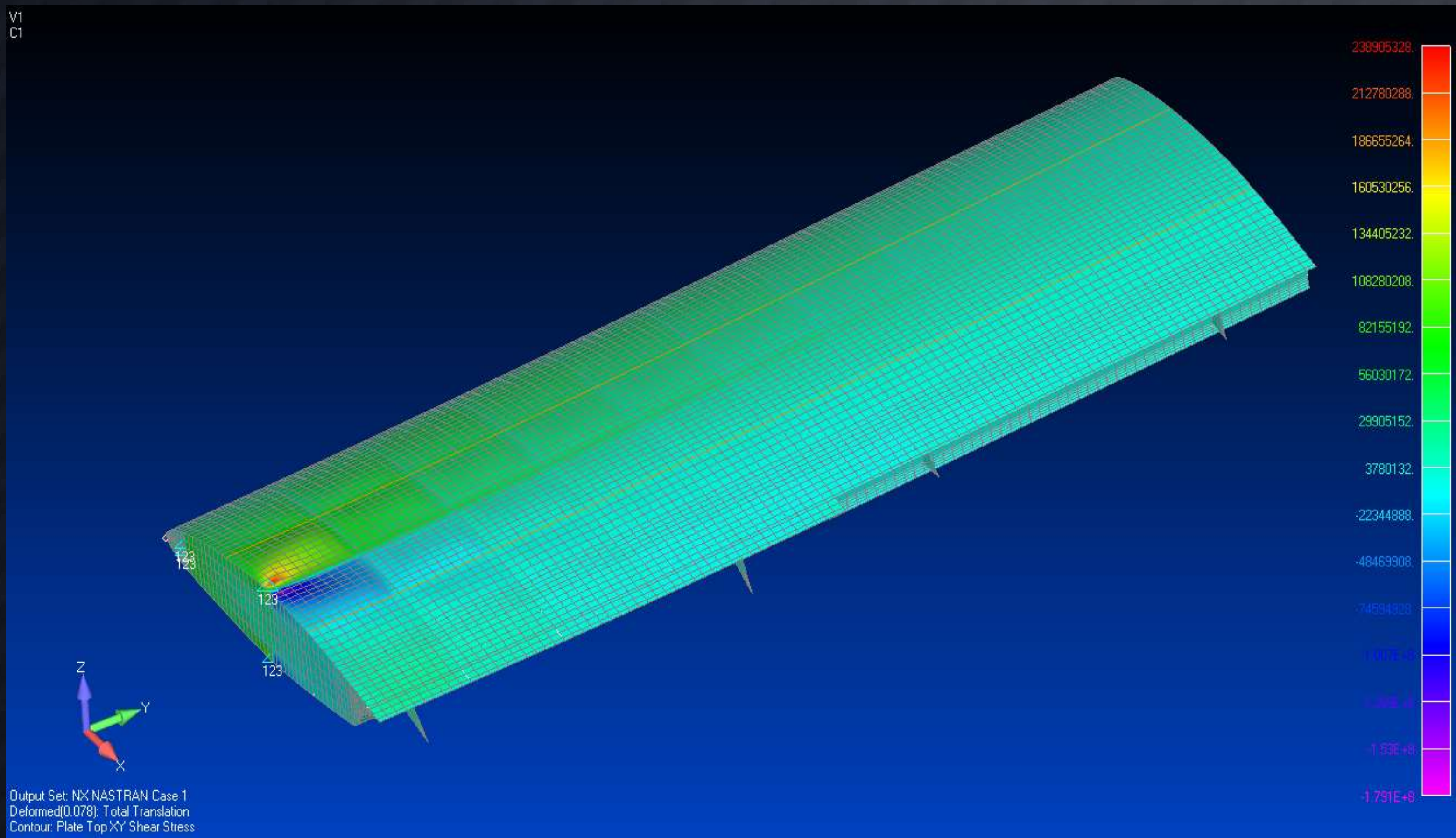
CS 1: Damaged wing-fuselage attachment

CS 1 - Damaged wing-fuselage attachment



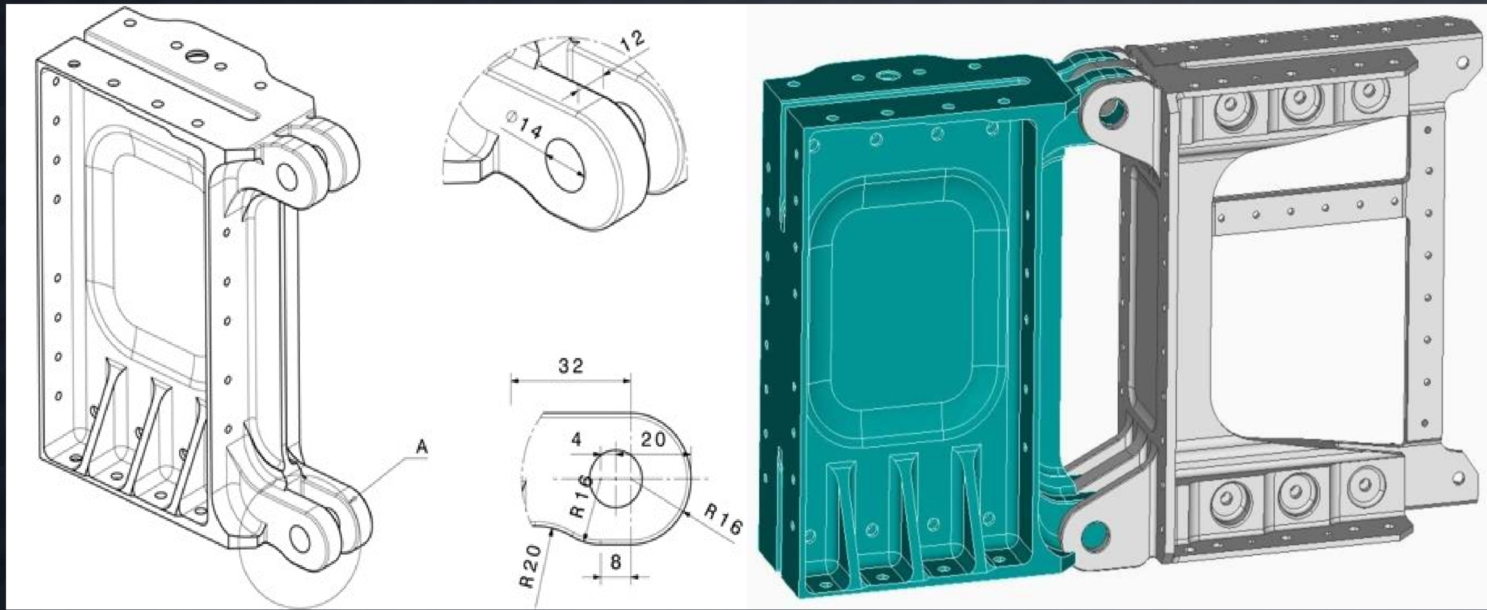
Aircraft fuselage stress analysis
(thin-walled structure analysis in Femap and NX Nastran)

CS 1 - Damaged wing-fuselage attachment

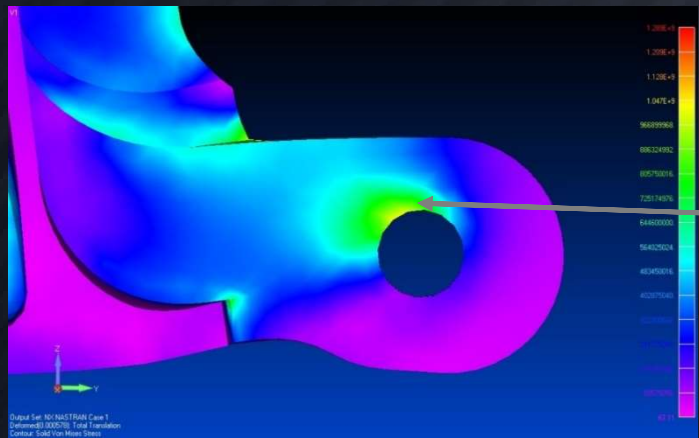


Wing stress analysis
(Femap and NX Nastran analysis)

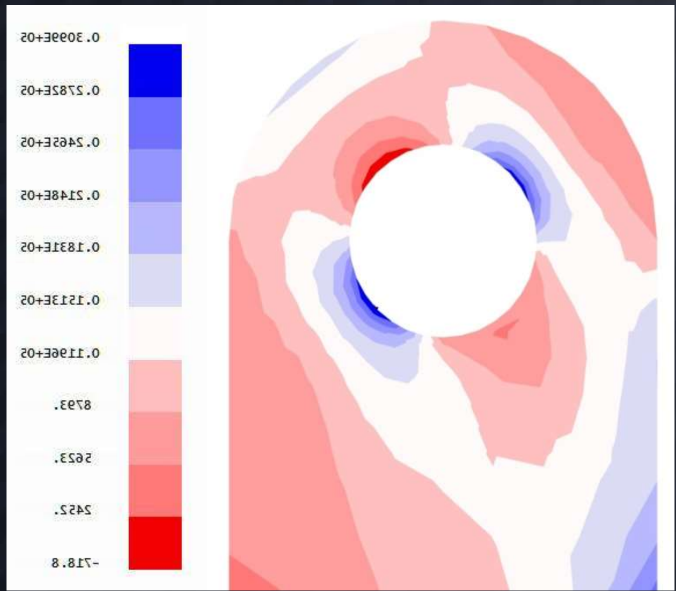
CS 1 - Damaged wing-fuselage attachment

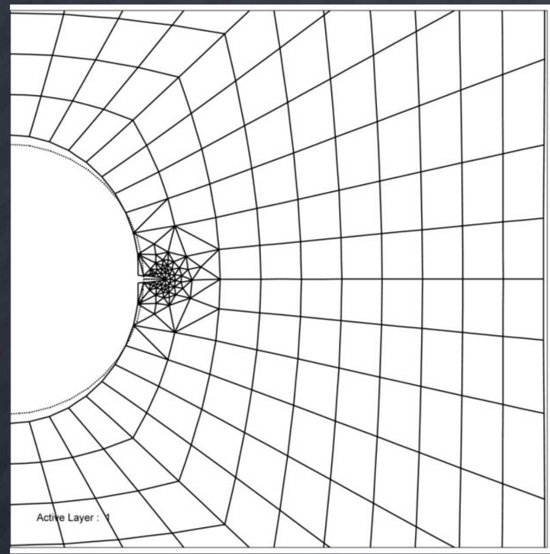


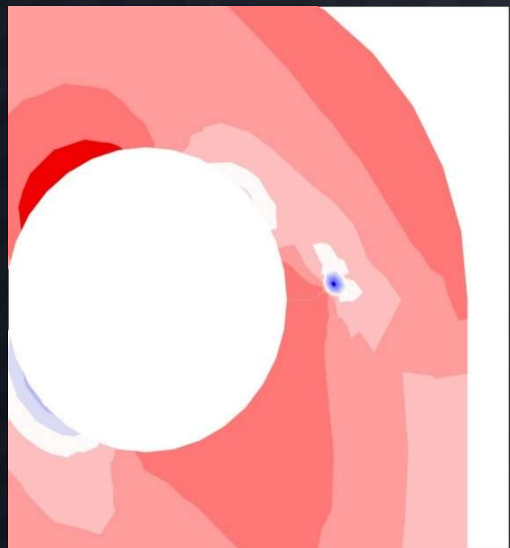
Wing-fuselage attachment

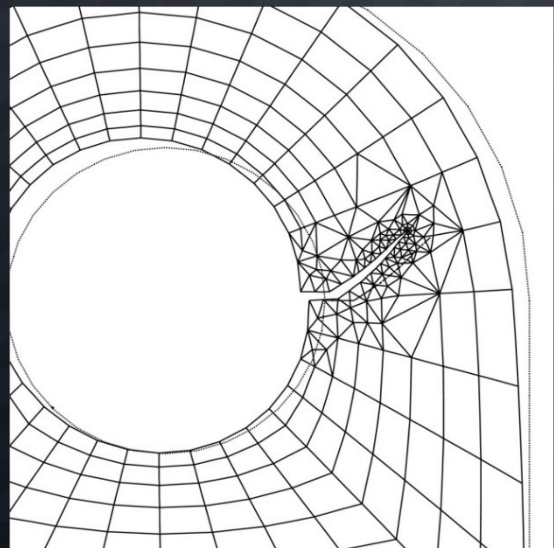


CS 1 - Crack growth in the lug (FRANC2D)



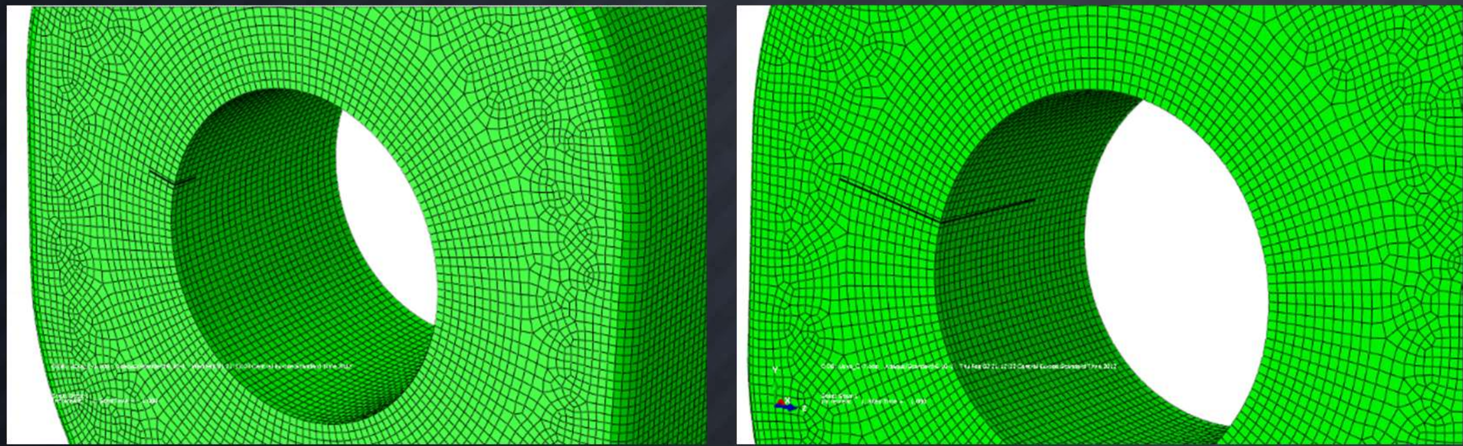




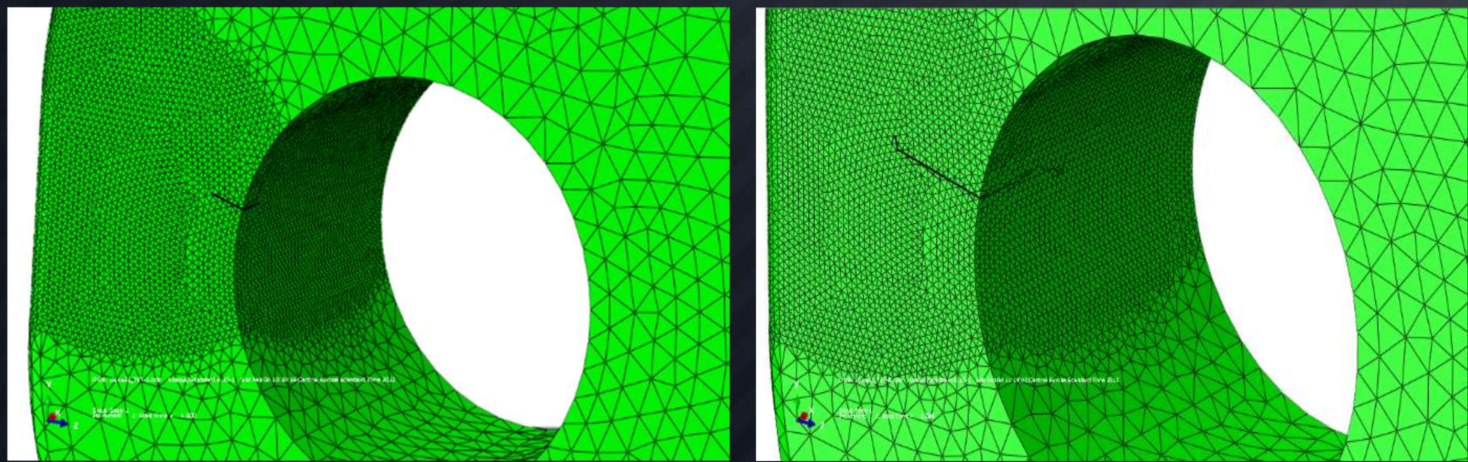


LUG CRACK PROPAGATION (FRANC 2D)

CS 1 -Crack growth in the lug (Abaqus XFEM)

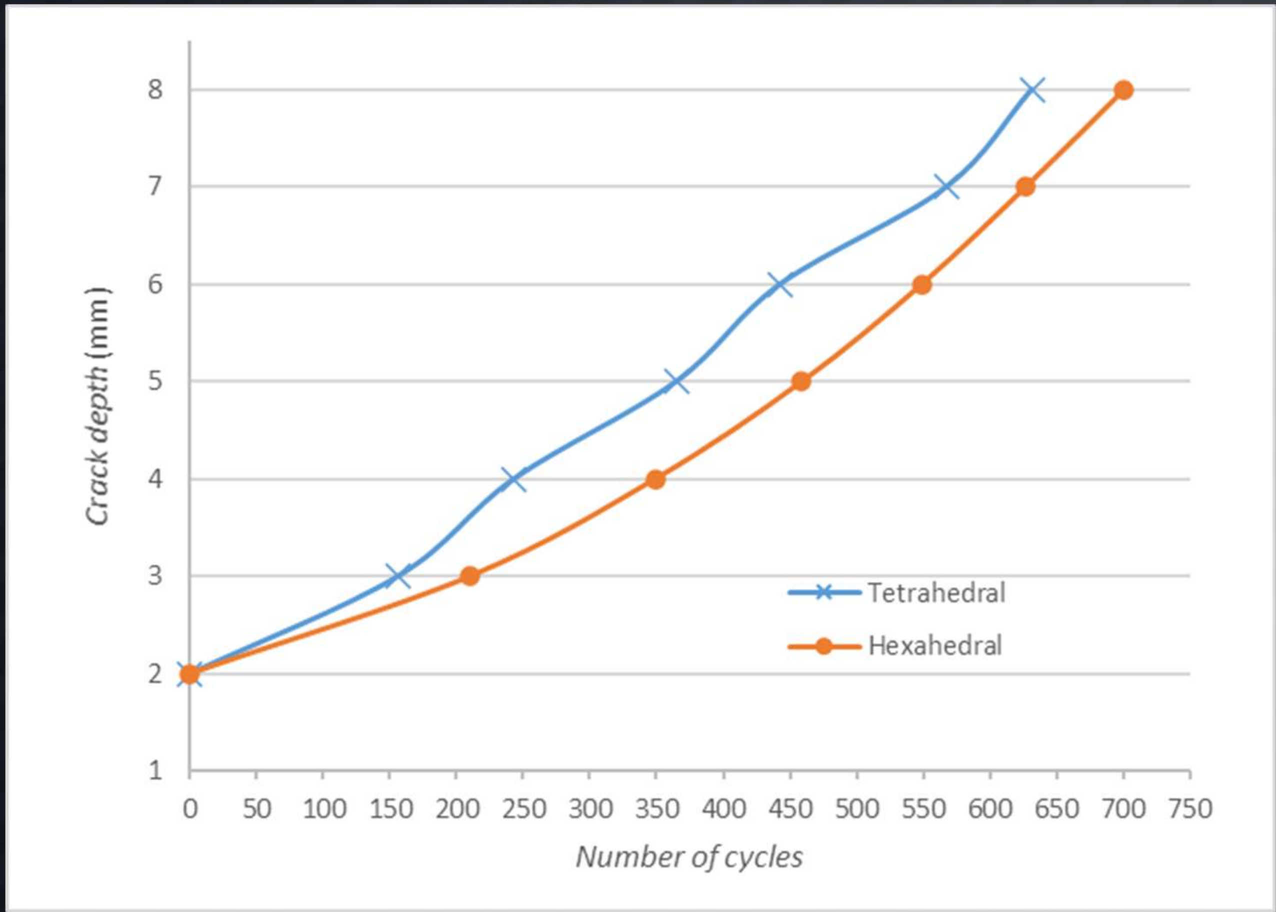


Initial penny-shaped crack in hexahedral mesh, and crack after 6th step of propagation



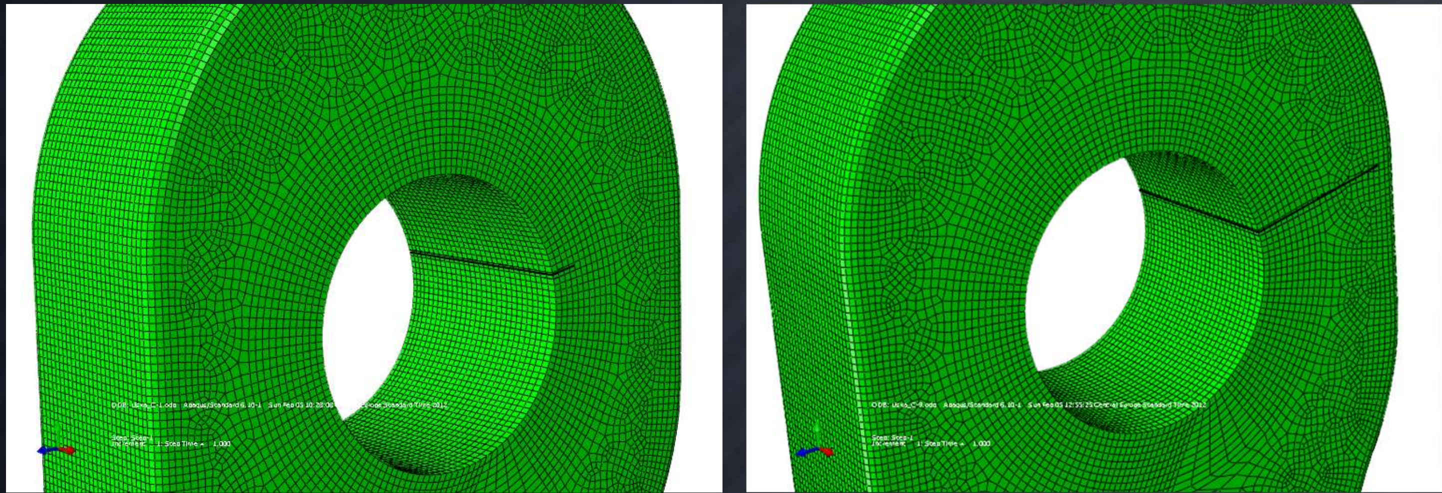
Initial penny-shaped crack in tetrahedral mesh, and crack after 6th step of propagation

CS 1 - Crack growth in the lug (Abaqus XFEM)

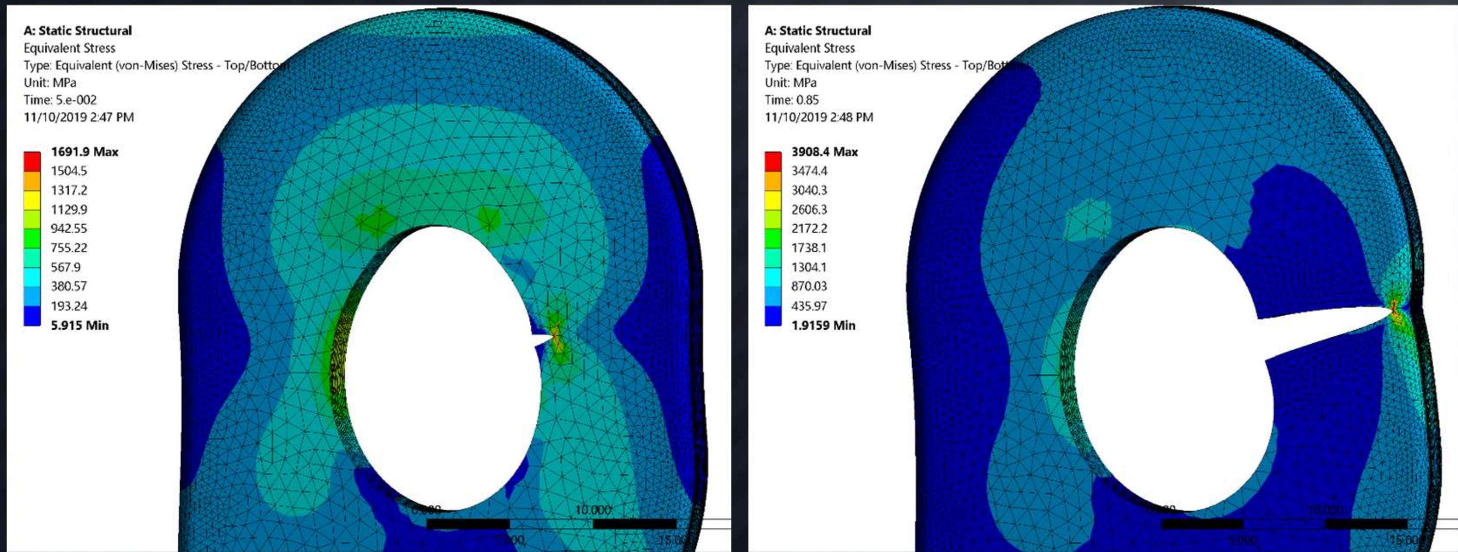


Number of cycles vs. crack depth for hexahedral and tetrahedral mesh

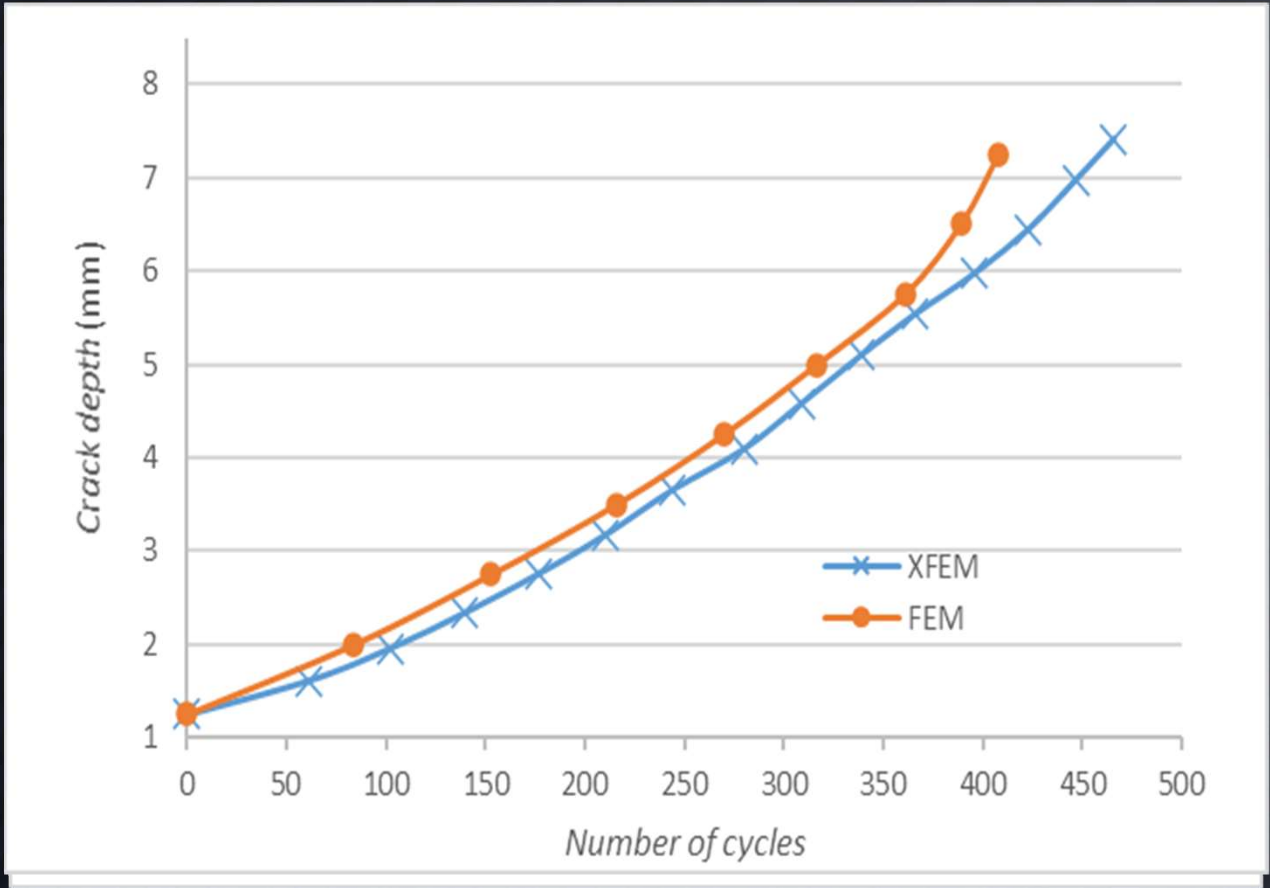
CS 1 - Through crack (XFEM vs FEM)



Initial through crack, and crack at the end of propagation

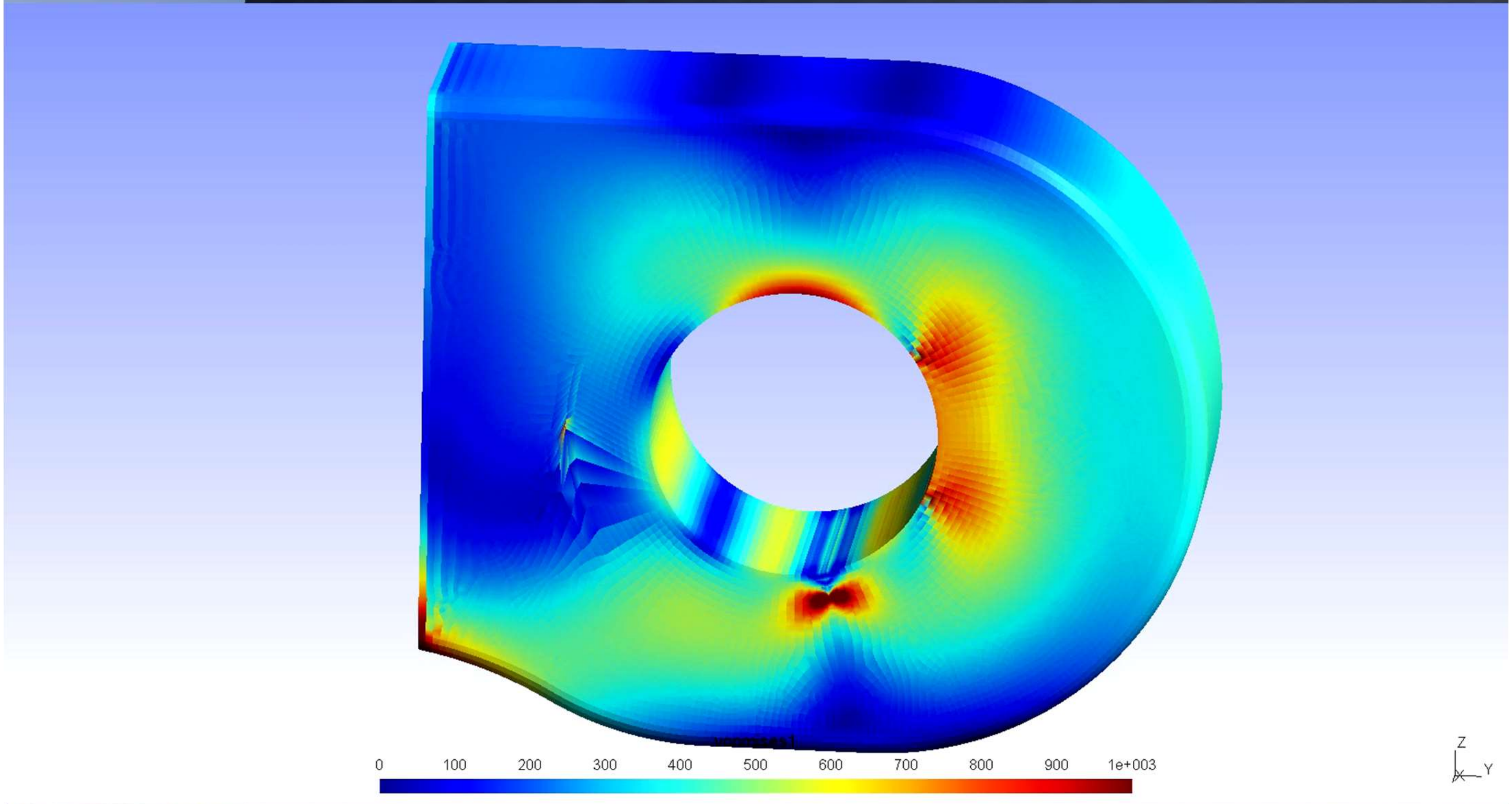


CS 1 - Through crack (XFEM vs FEM)

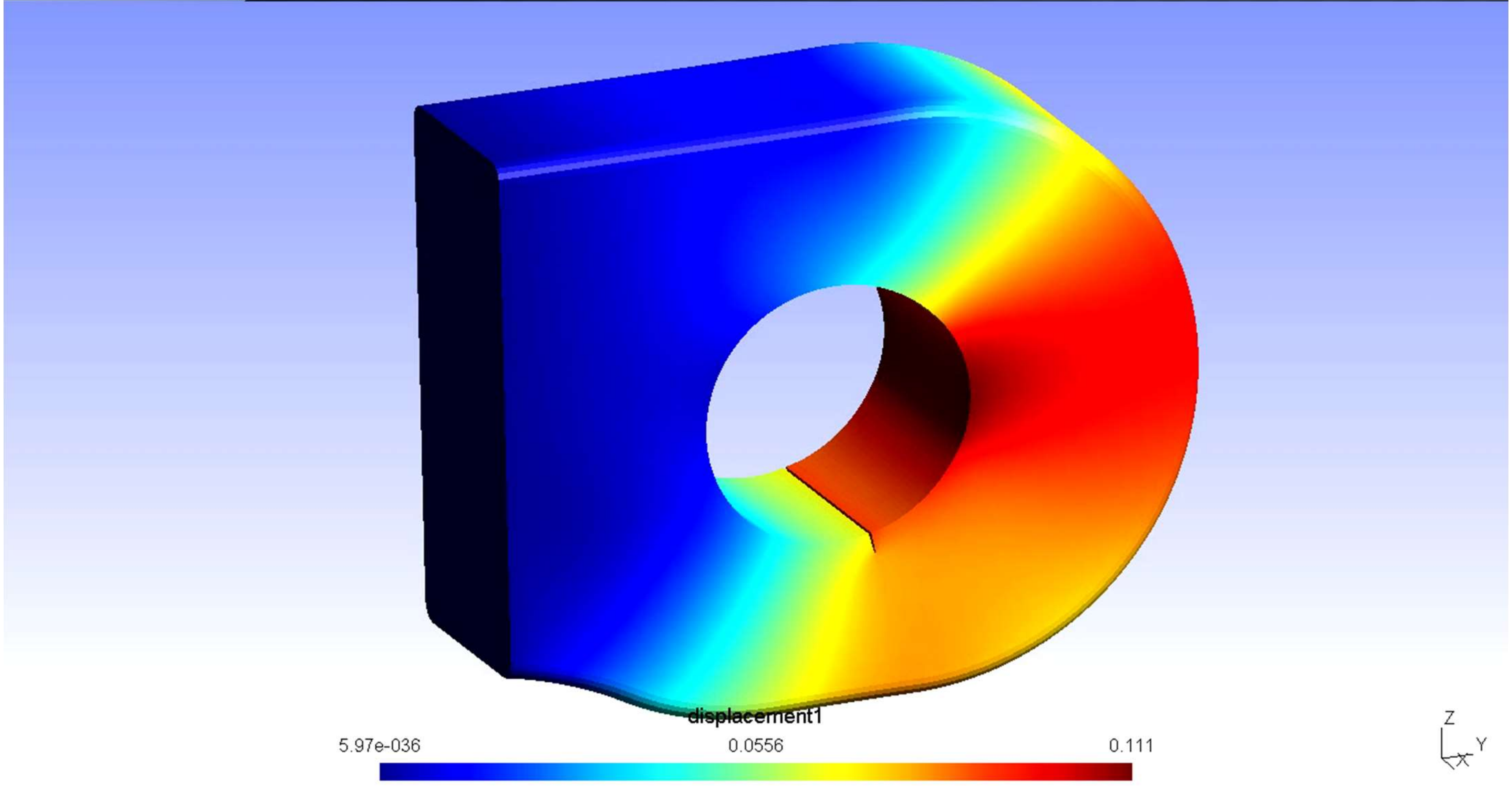


Comparison of values

CS 1 - Lug stress with growth (XFEM)

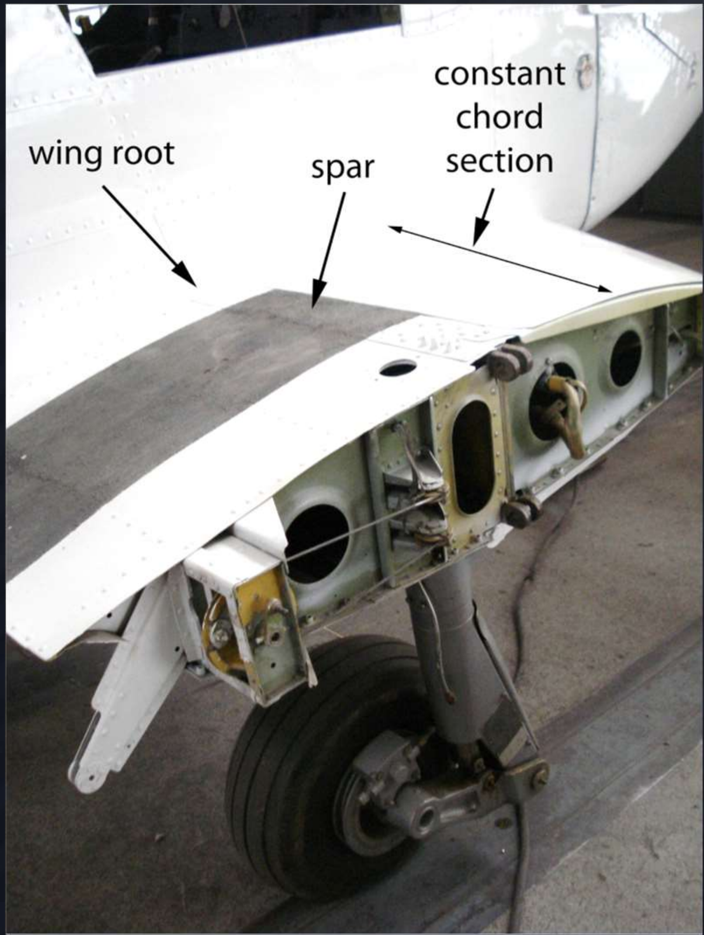


CS 1 - Lug displacement (XFEM)

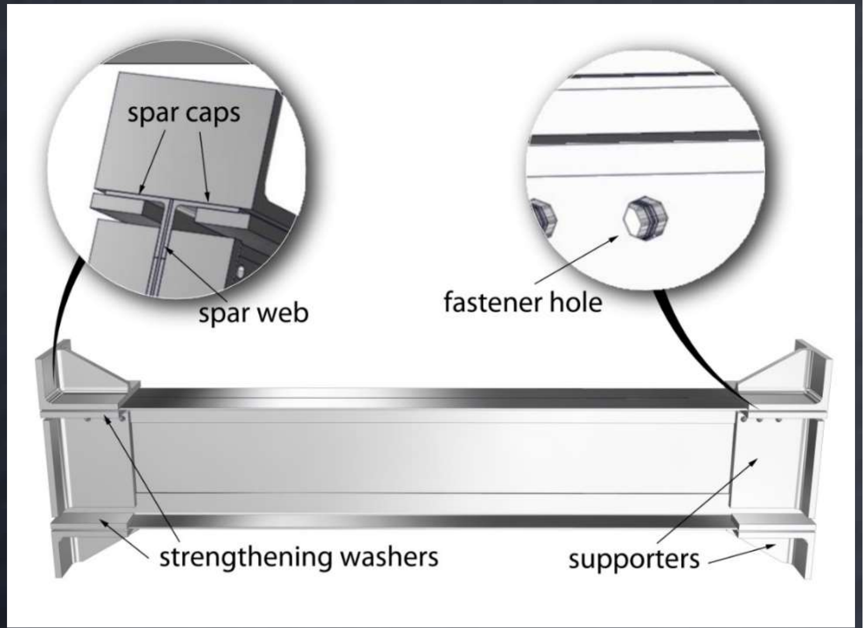


CS 2: Crack growth in the wing spar

CS 2 – Fatigue life of the wing spar (2024-T3)

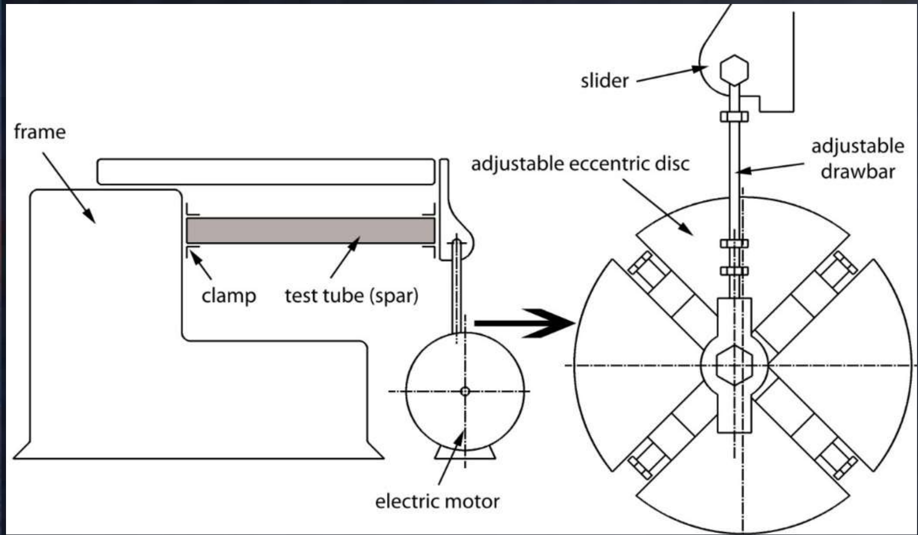


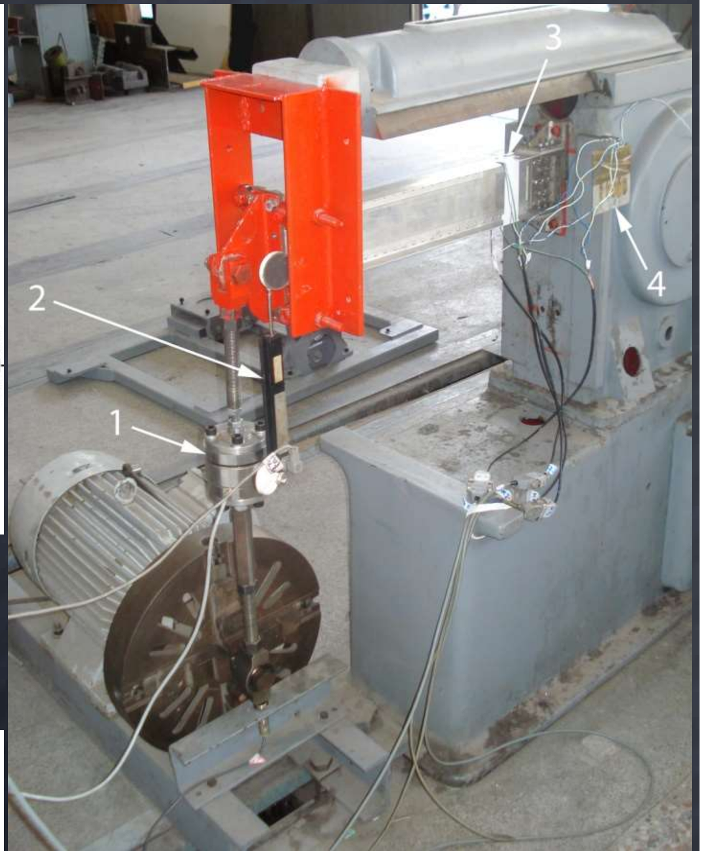
Wing root assembly of light aircraft



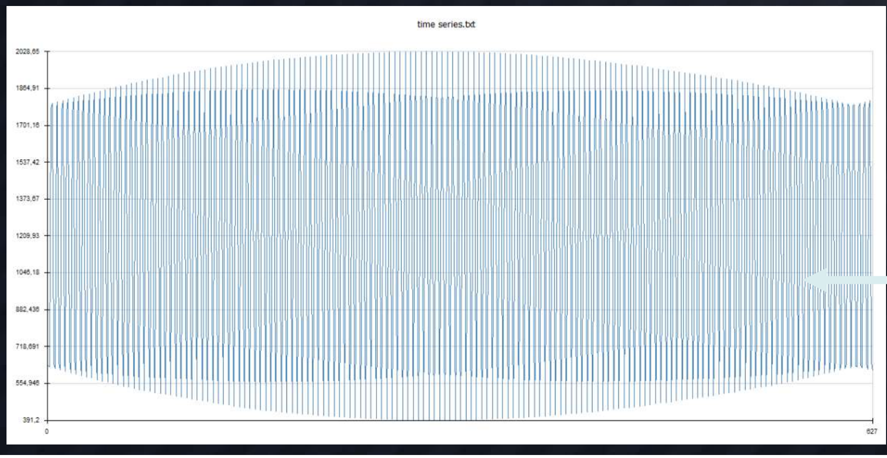
The spar and supporting elements used in experiment

CS 2 – Experimental setup for fatigue testing



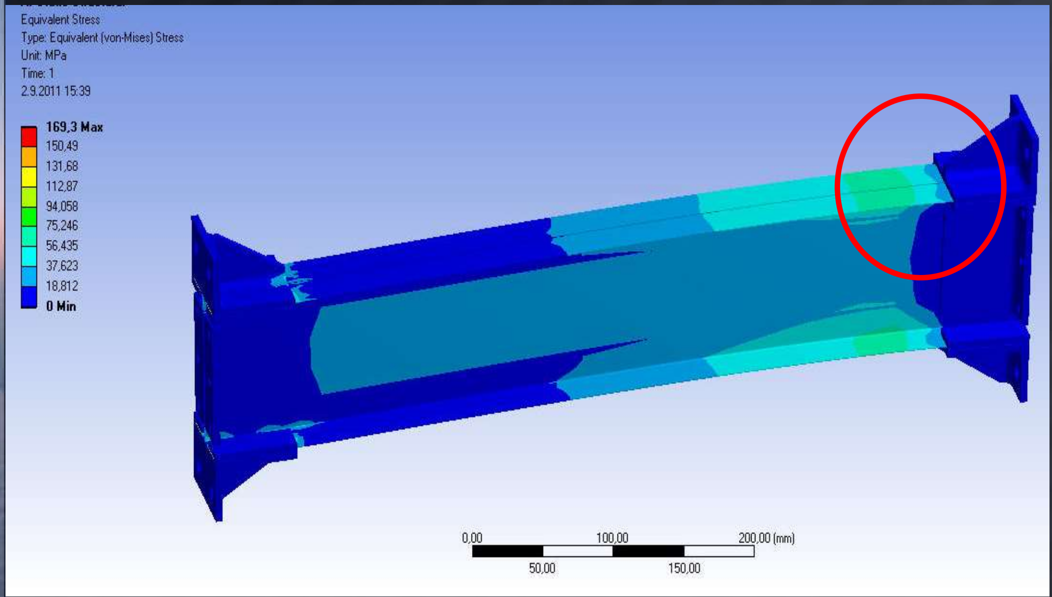


Fatigue system used in testing of light aircraft spar



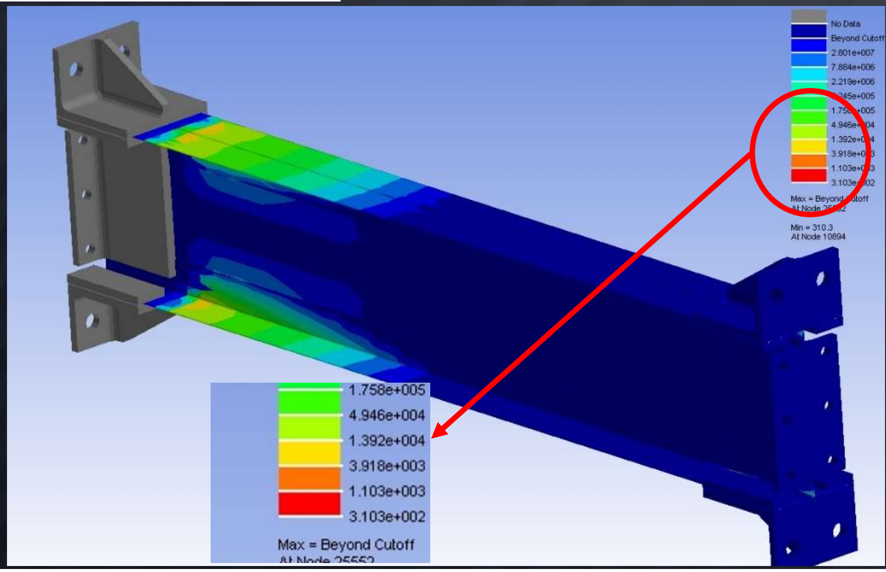
Narrow band random loading used in the experiment and FEA

CS 2 – FEM identification of the critical area

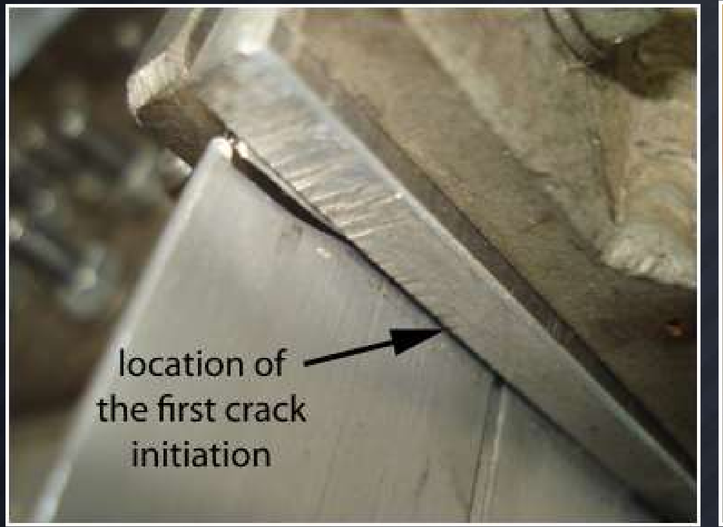


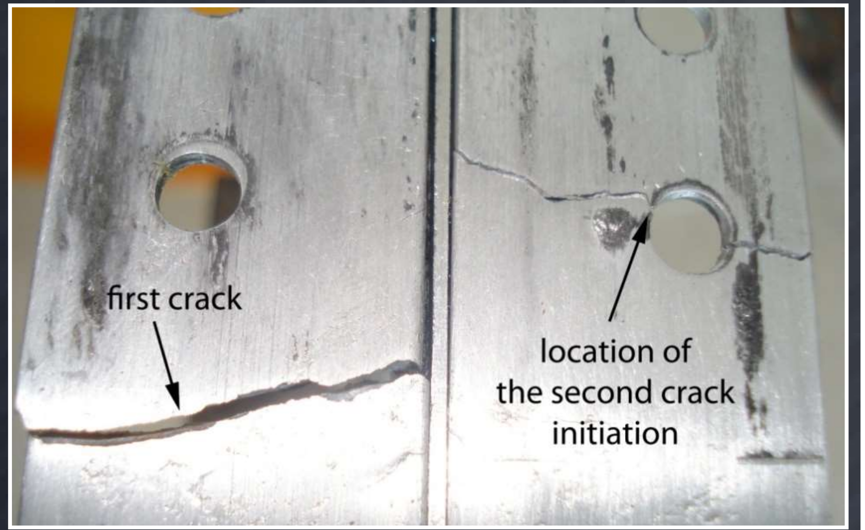
The most likely area of the crack appearance is where high tensile stress occurs

Estimated life to crack occurrence is 7944 cycles (min value at node)



CS 2 – Cracks' initiation and growth

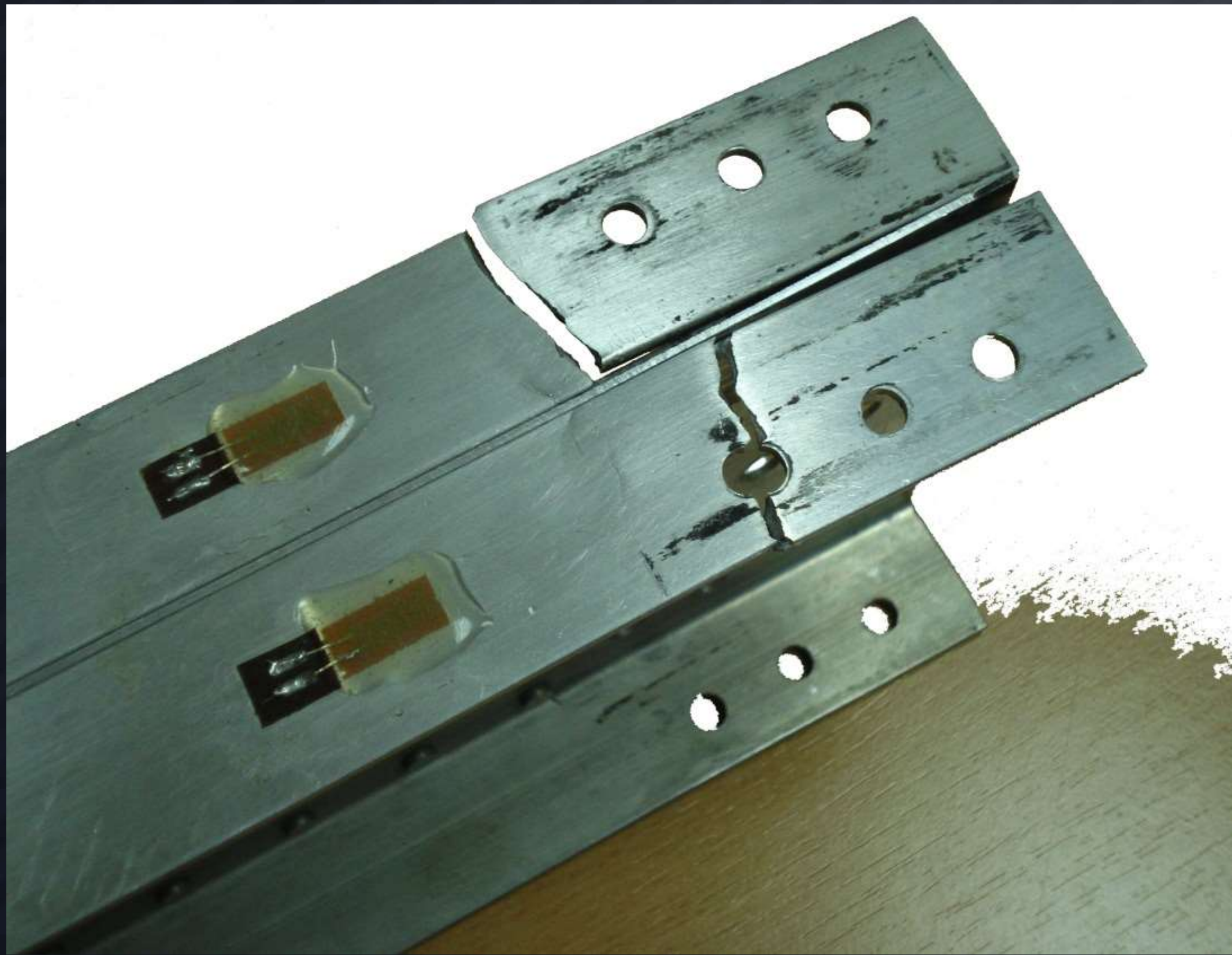




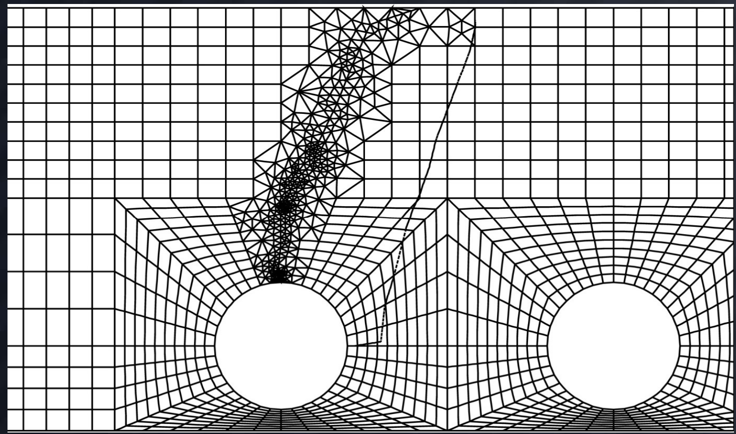
Locations and paths of cracks on horizontal walls of the spar caps

Cracks' growth paths on the vertical walls of the left and right spar cap

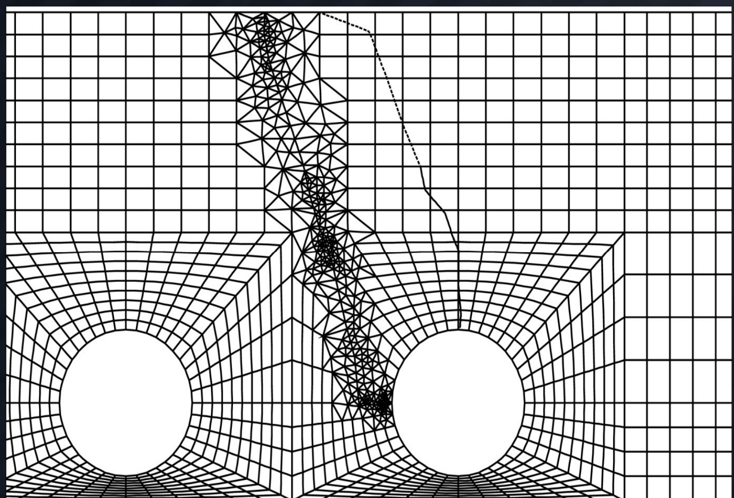
CS2 – The spar after experiment



CS2 – FEM simulation in FRANC2D

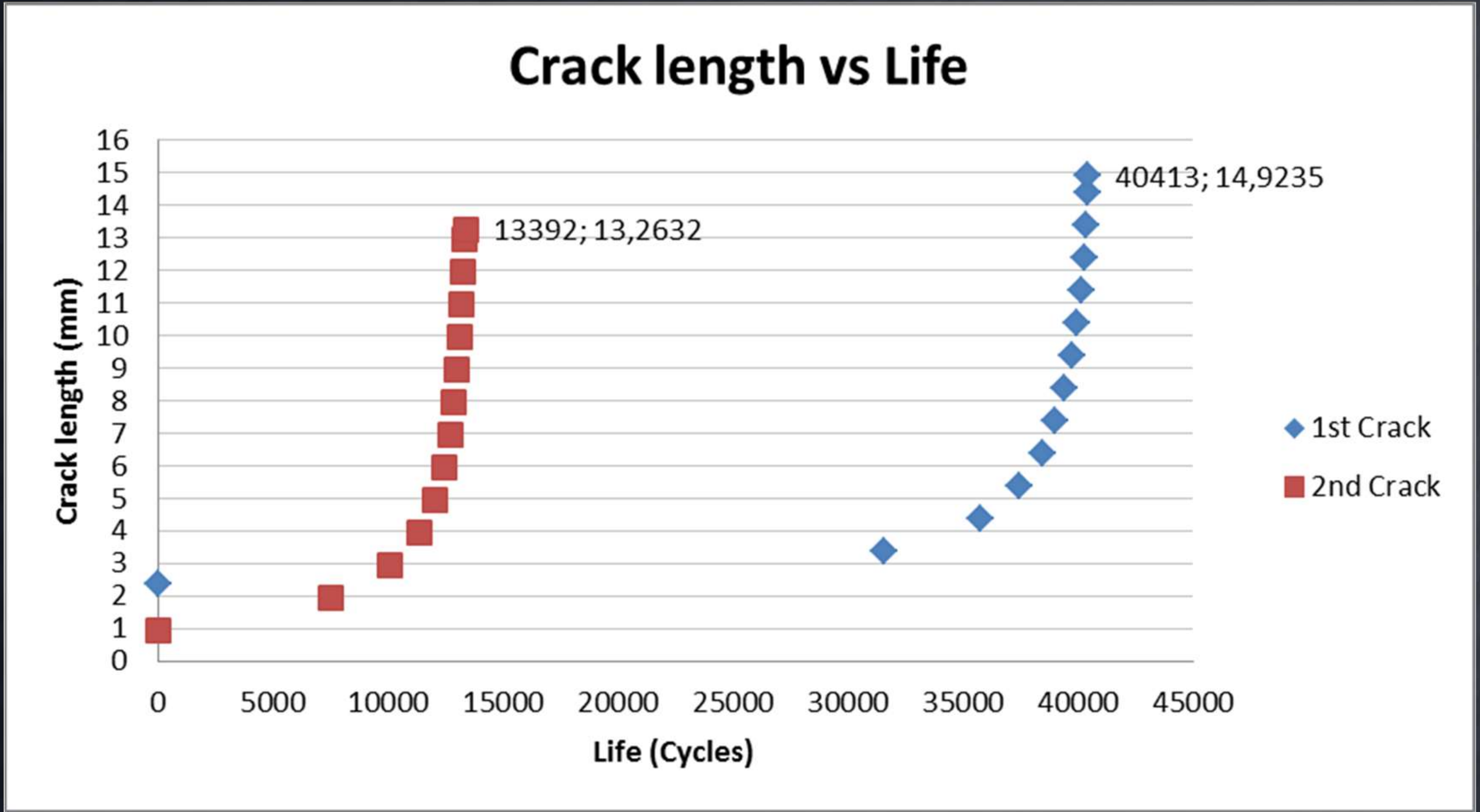


1st and 2nd crack path obtained in FRANC2D



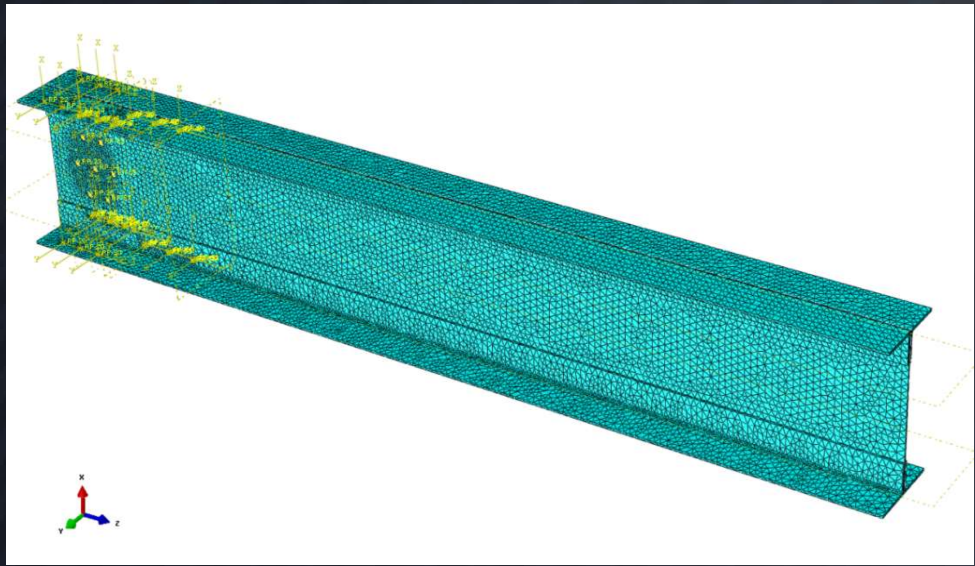
Cracks' paths observed in experiment

CS 2 – Estimated fatigue life

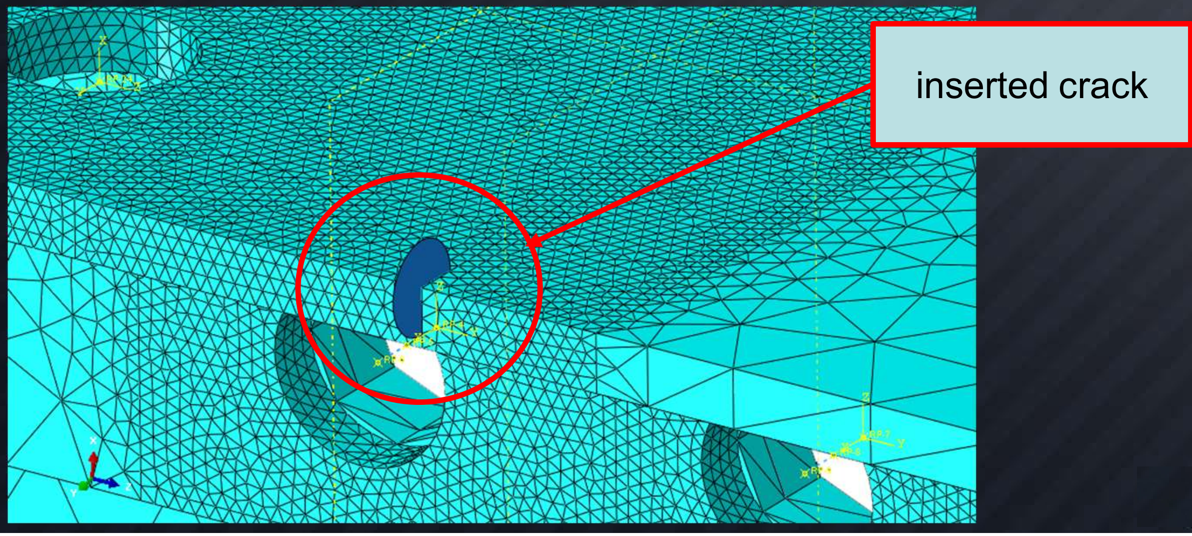


Graph Crack length vs. Life obtained after integration of NASGRO formula

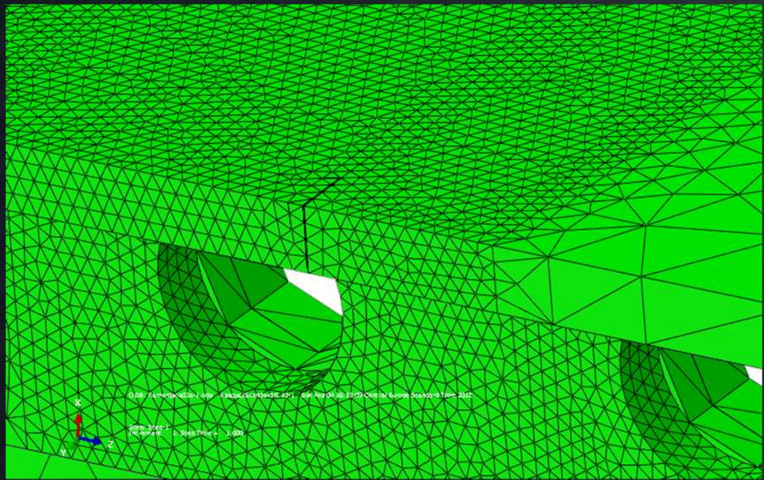
CS 2 – XFEM calculation (Abaqus)



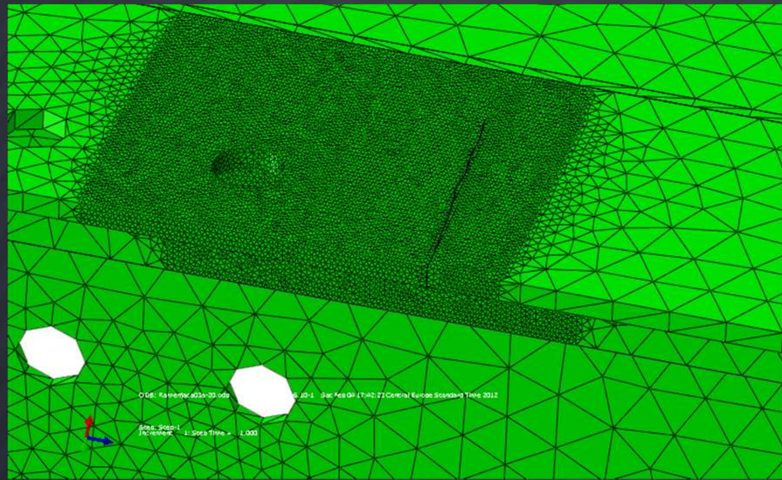
FE model of spar (applied displacement 3mm at free end)



CS 2 – Crack growth on horizontal cap wall



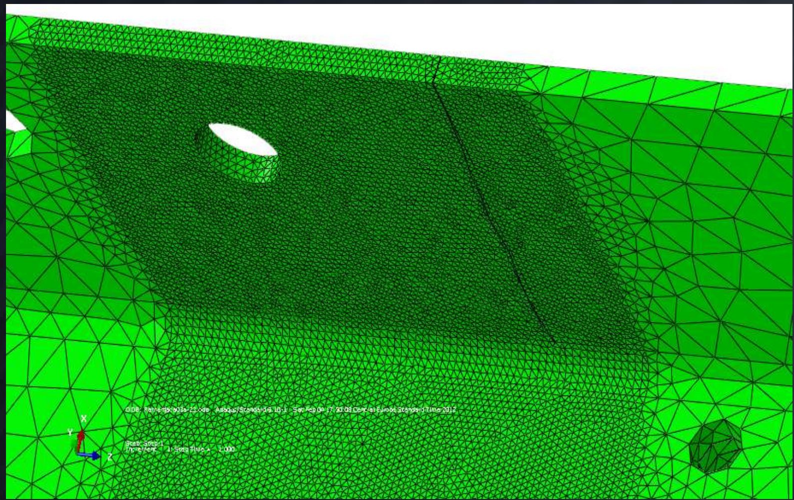
1st step (crack „opening“)



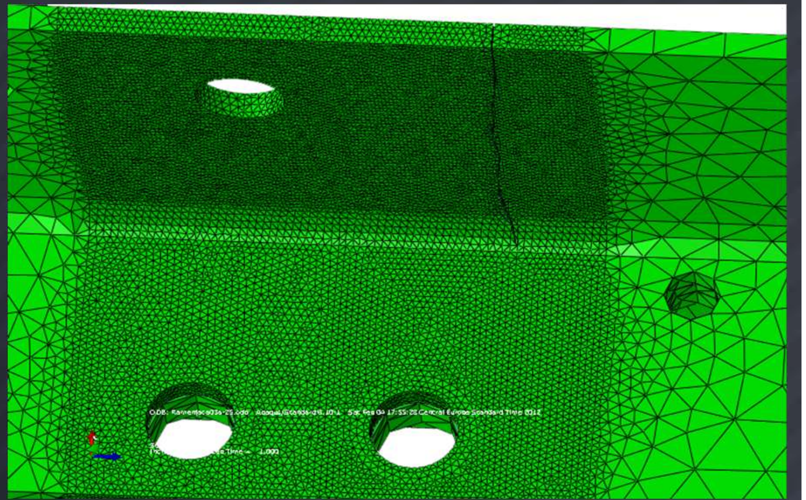
Crack shape after 19th step of propagation

Real crack on the spar cap

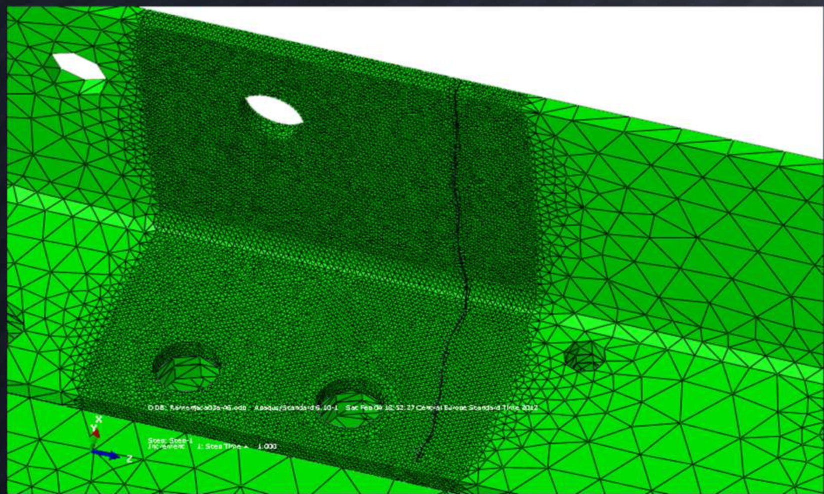
CS 2 – Crack growth on vertical cap wall



Crack shape after 22nd step of propagation

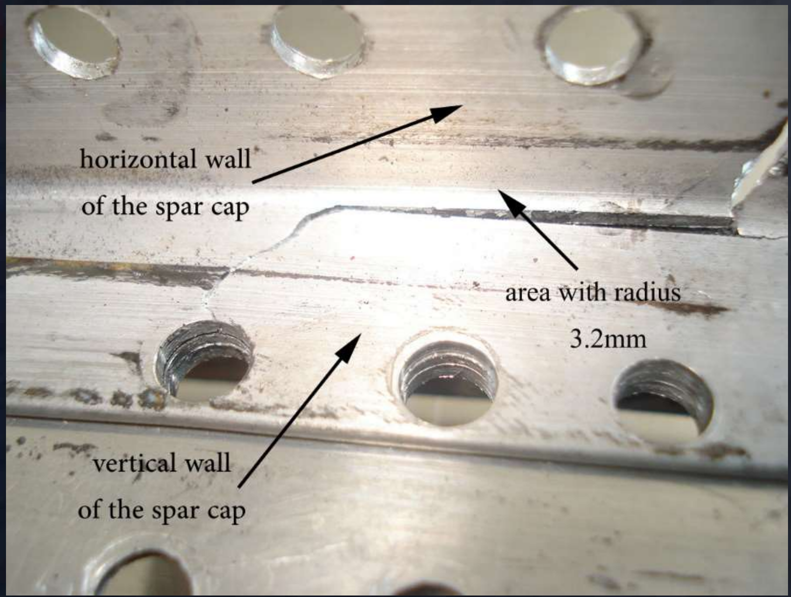


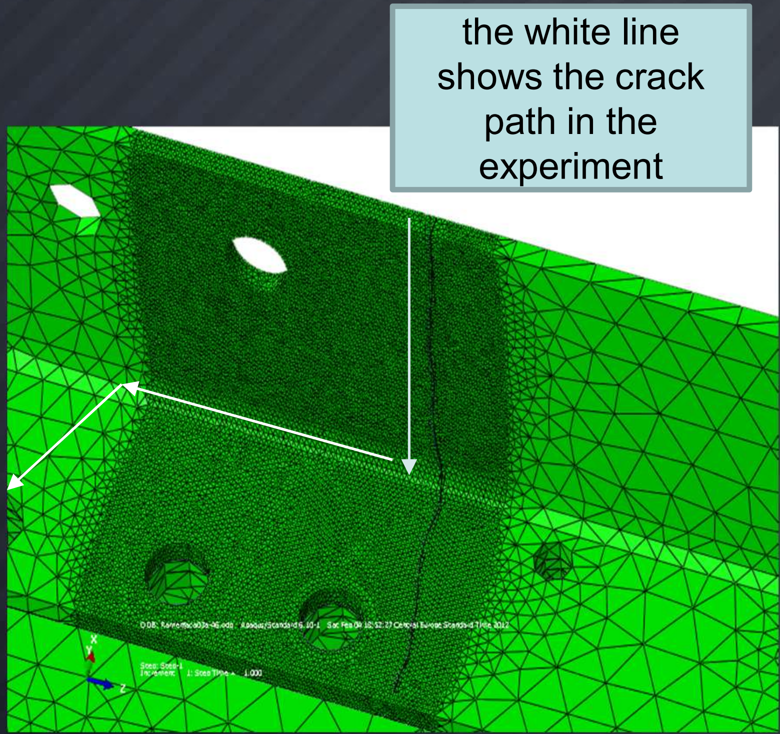
24th step of propagation



Crack shape after 45th step of propagation

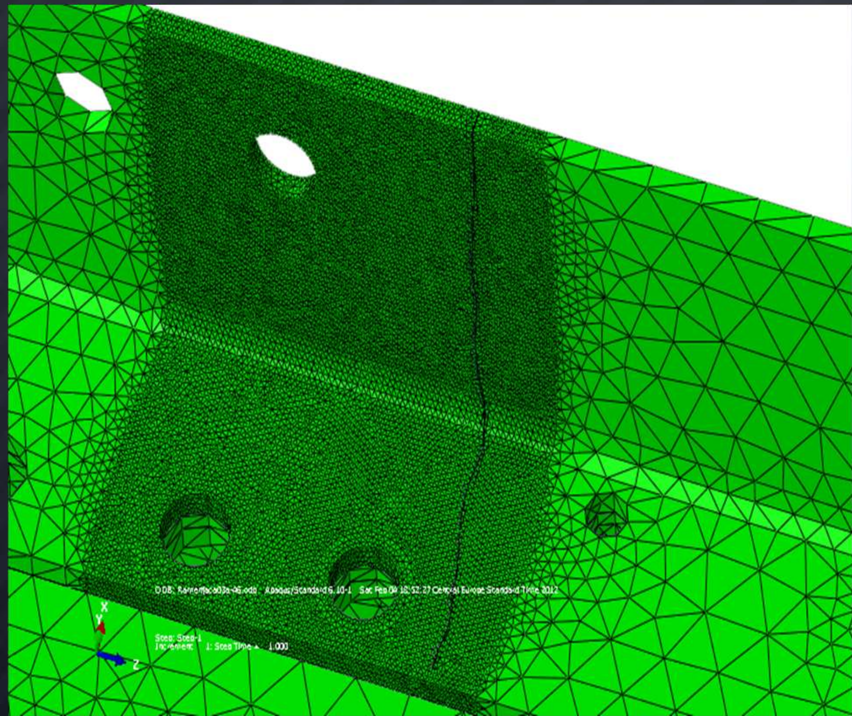
CS 2 – Comparison with experiment





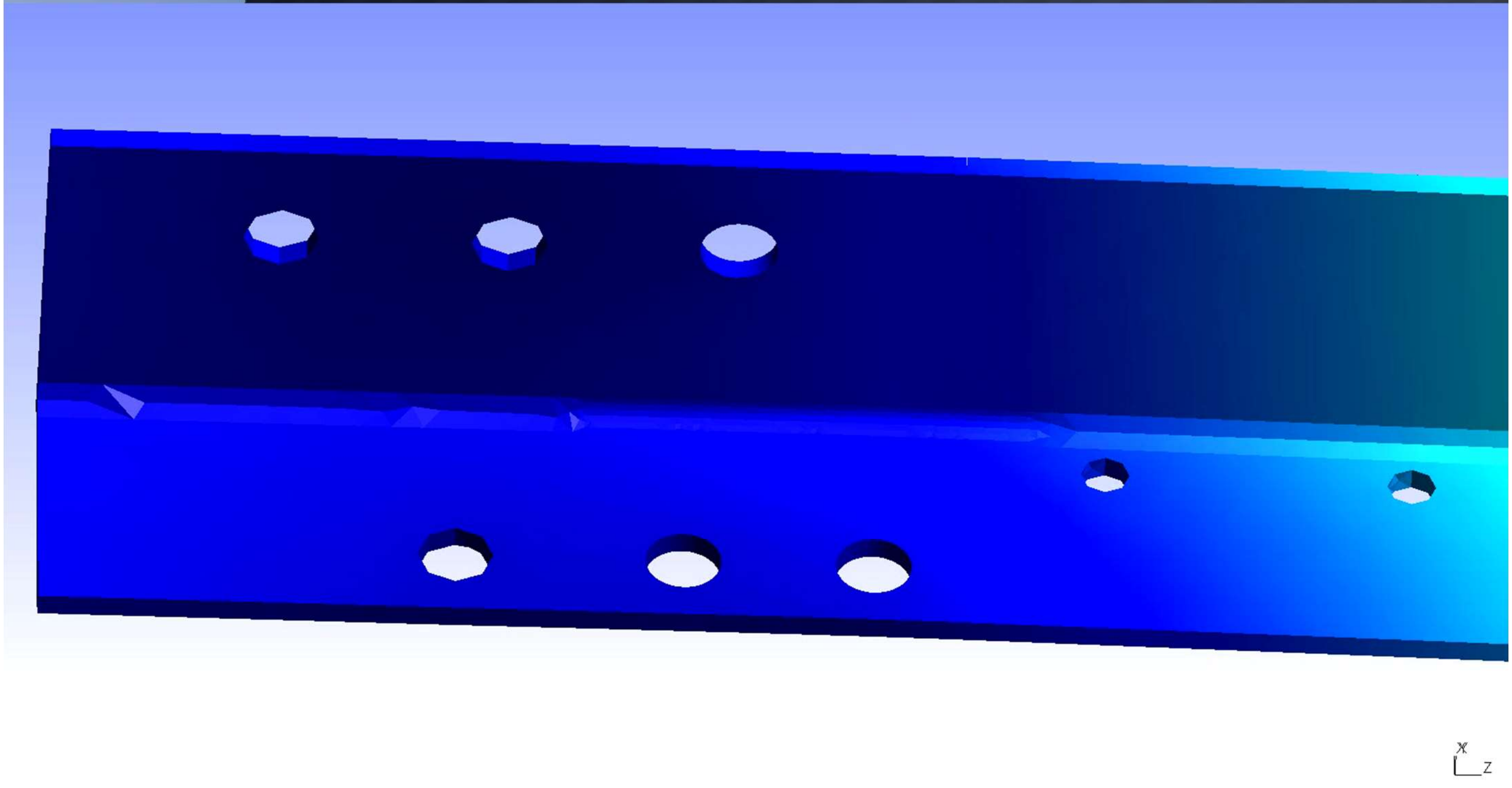
The crack path on the spar cap in the experiment (left) and simulated crack path (right)

CS 2 – Comparison with experiment

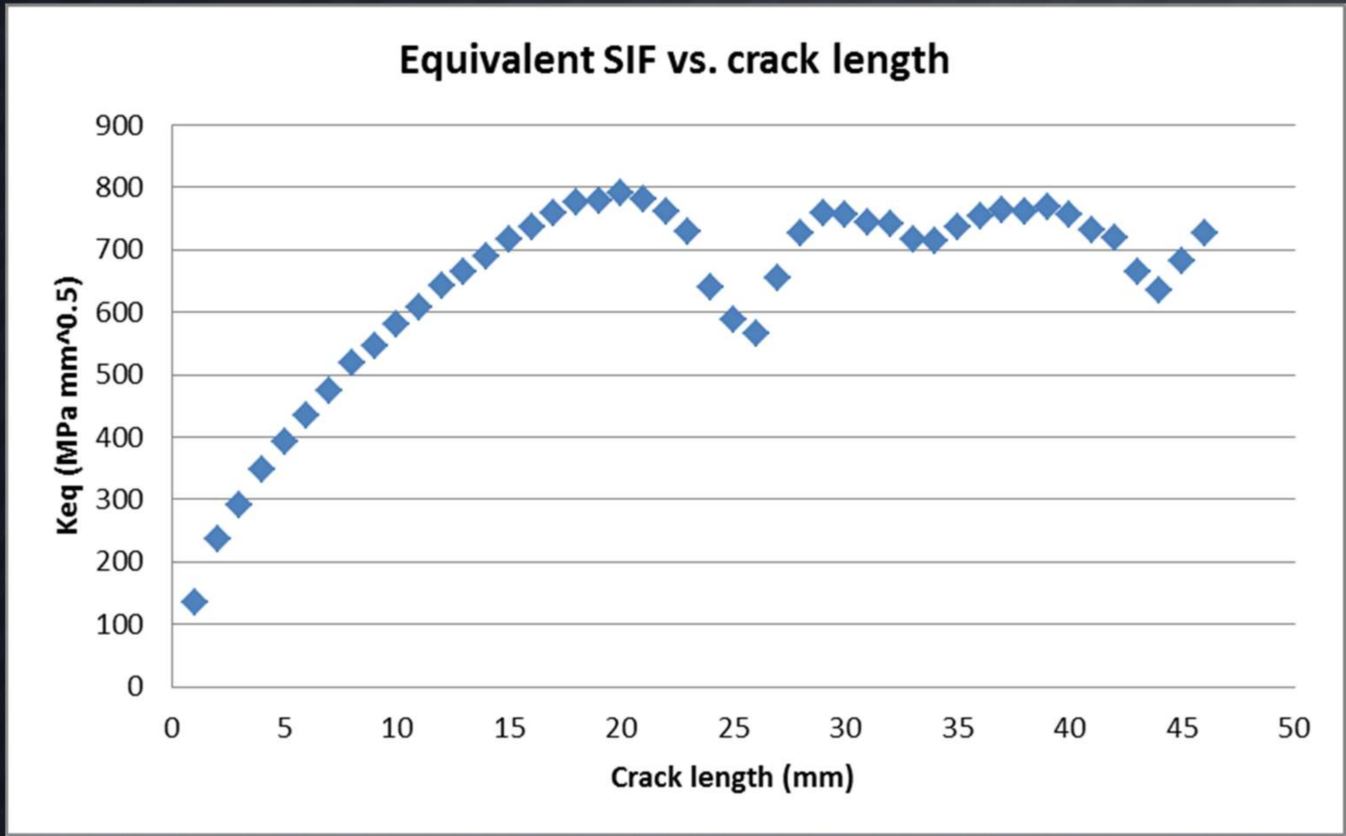


View of the crack on the cap (left) after stabilization (residual stress removal) and simulated crack path (right)

CS 2 – Crack growth (displacement)

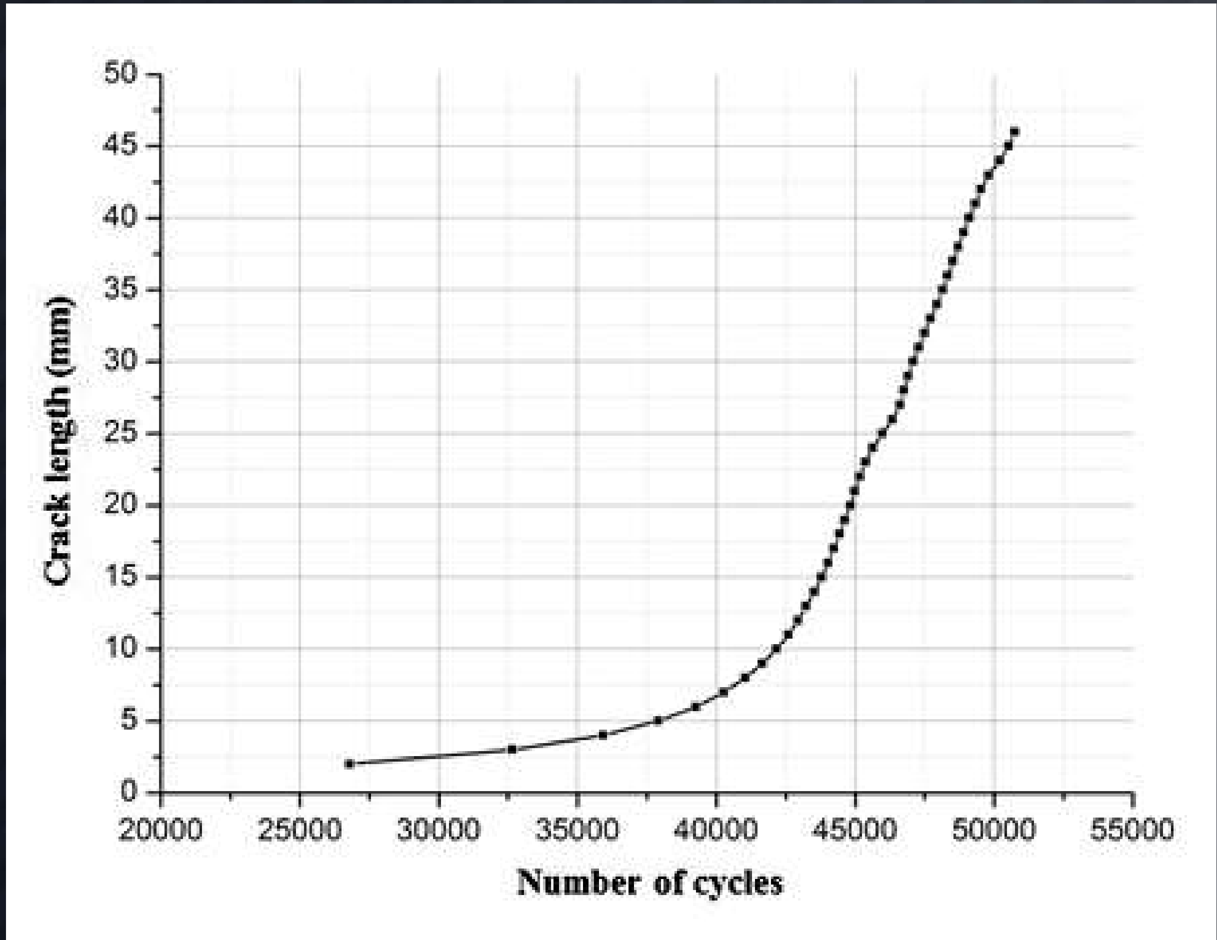


CS 2 – Calculated SIF values (Abaqus)



Values of equivalent stress intensity factor (K_{eqv}) as a function of crack length

CS 2 – Fatigue crack growth life



Crack length as a function of the number of cycles (Paris coefficients: $C = 6.106 \times 10^{-11}$, $n = 2.60$)

CS 2 – Comparison of results

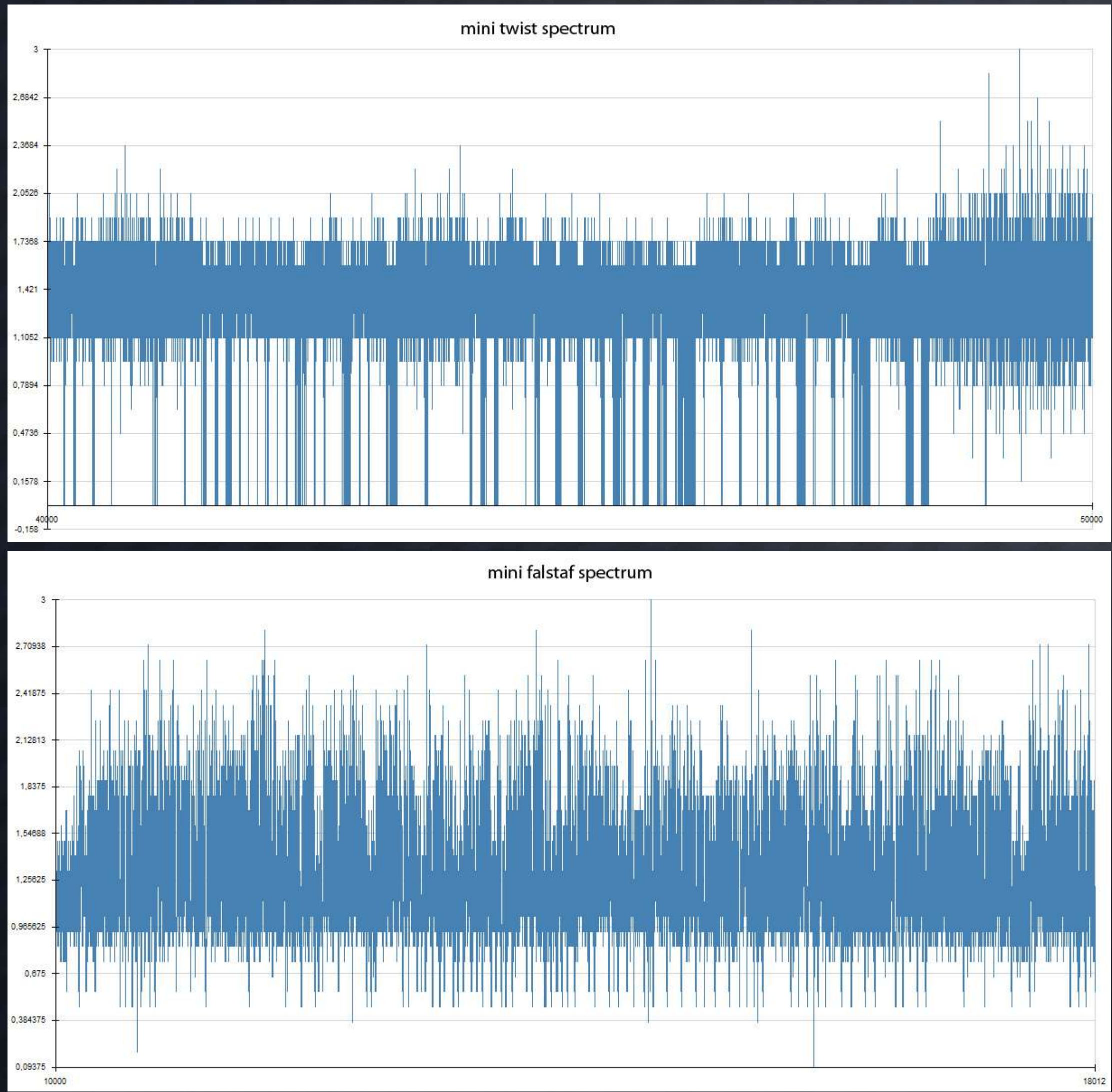
Ansys (crack occurrence)	FRANC2D (crack growth)		Total (2D)	Abaqus (crack growth)	Total (3D)
7944 cycles	1st crack	40,413 cycles	48,357 cycles	50,743 cycles	58,687 cycles
	2nd crack	13,392 cycles	21,336 cycles		

Fatigue life obtained using FEM and XFEM

Crack occurrence		Experiment stopped after
1st	8,452 cycles	58,520 cycles
2nd	39,450 cycles	

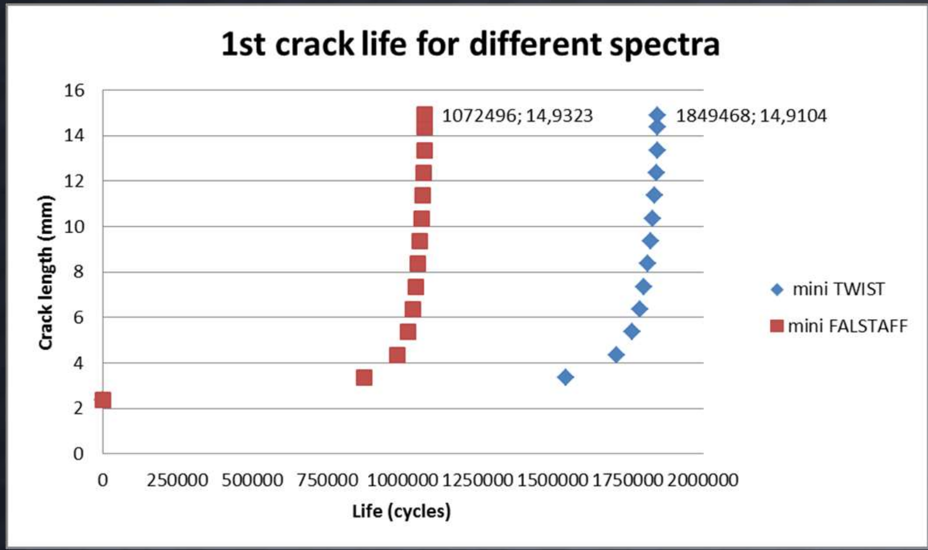
Fatigue life obtained in experiment

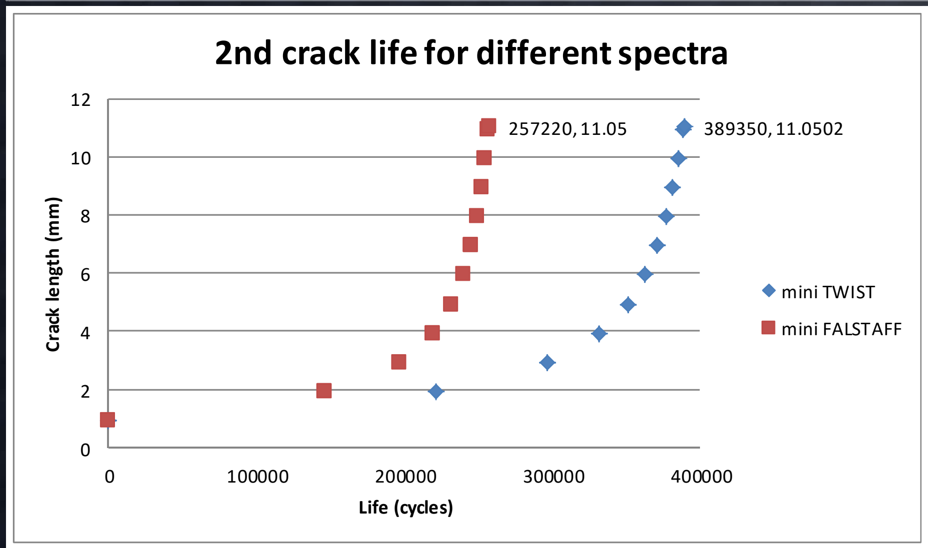
CS 2 – Standard spectra in aircraft design



Mini-TWIST spectrum and mini-FALSTAFF spectrum

CS 2 – Standard spectra and fatigue life

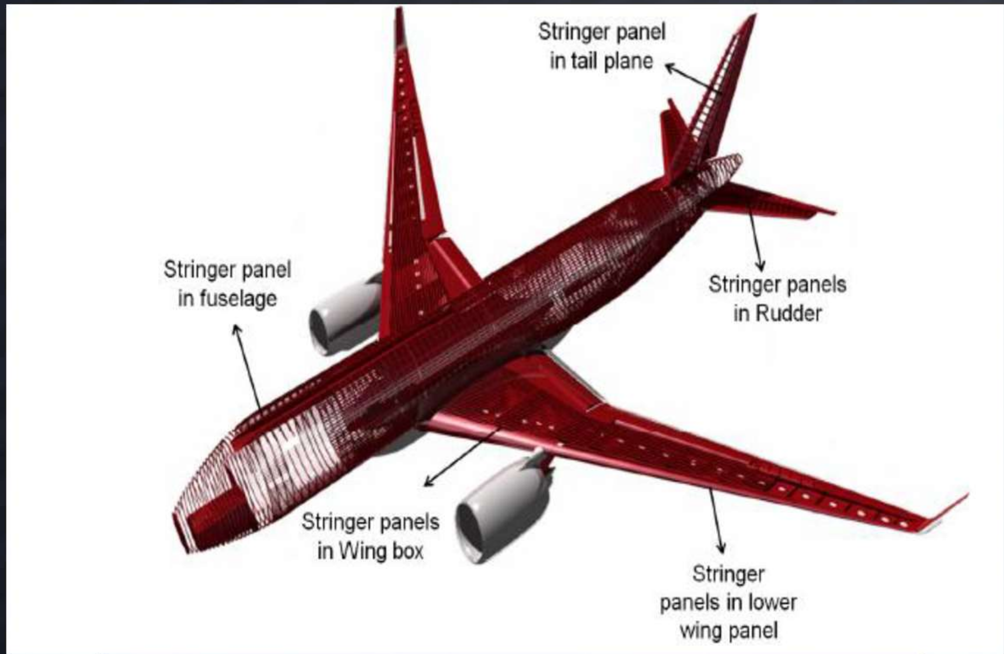




Crack length vs. Life under different spectra (based on FRANC2D results)

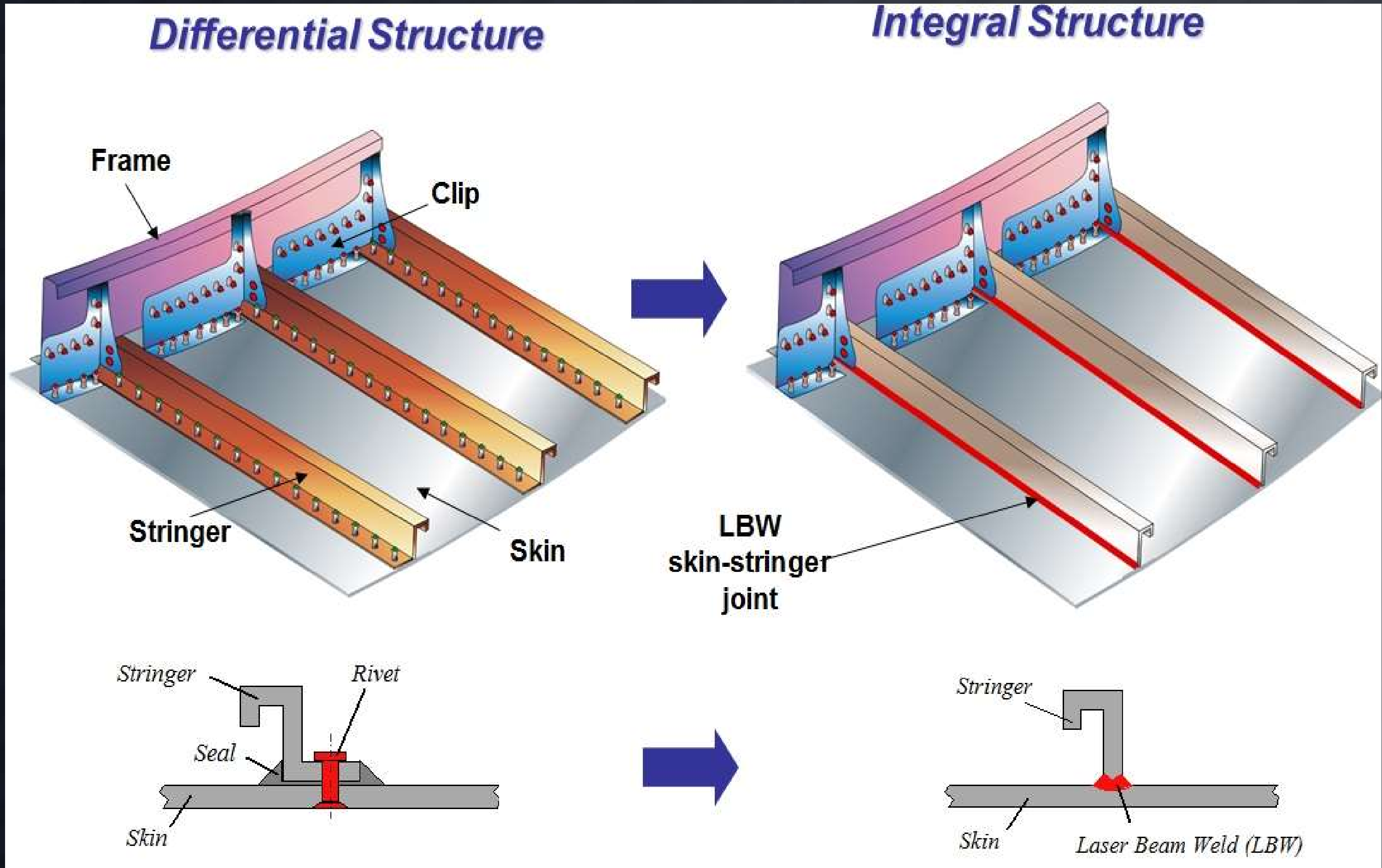
CS 3: Fatigue life assessment of damaged integral skin–stringer panel

CS 3 – Integral skin–stringer panels



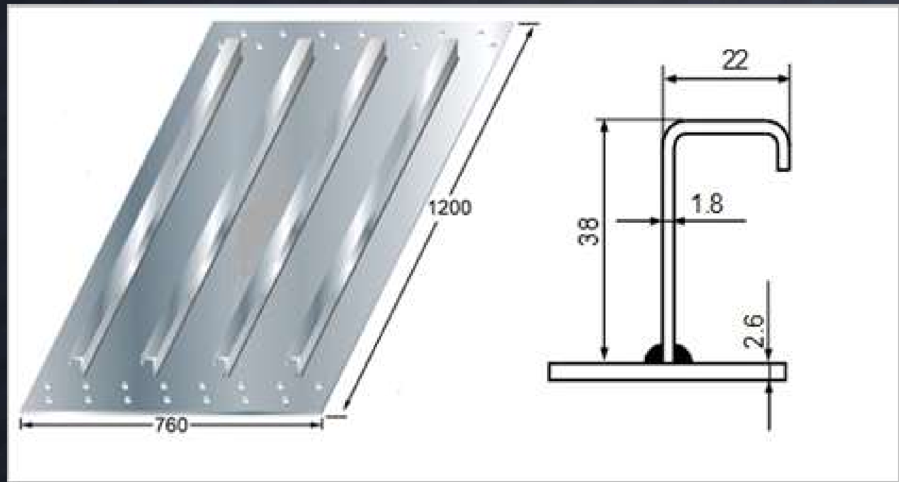
Skin-stringer panels are widely used in modern aircraft

CS 3 – Integral skin–stringer panels



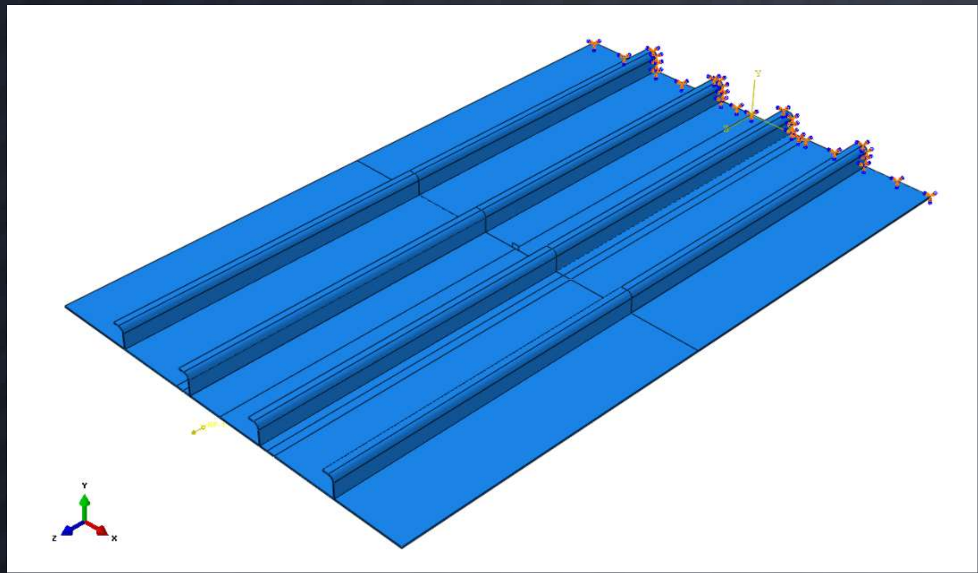
Differential structure vs. Integral structure

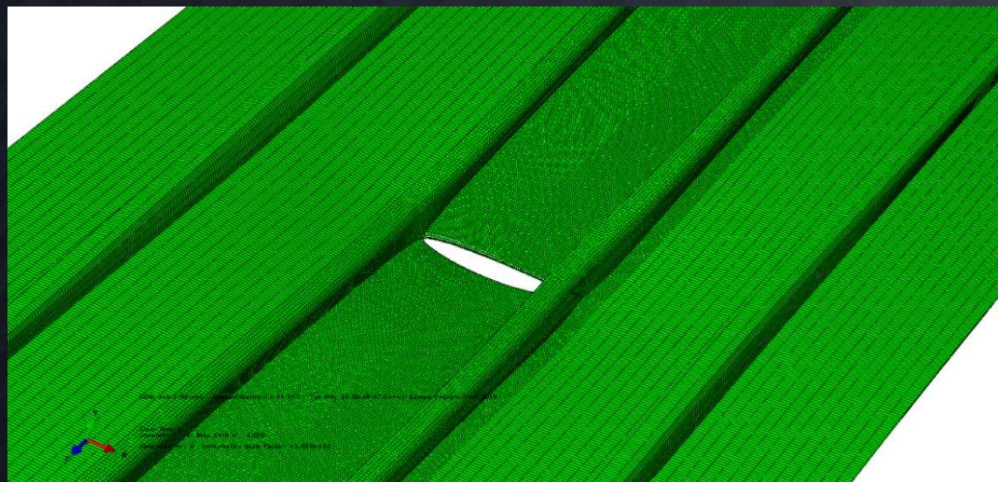
CS 3 – Experimental analysis of panels



4-stringer panel (6156-T6) was tested in GKSS research center (Hamburg, Germany)

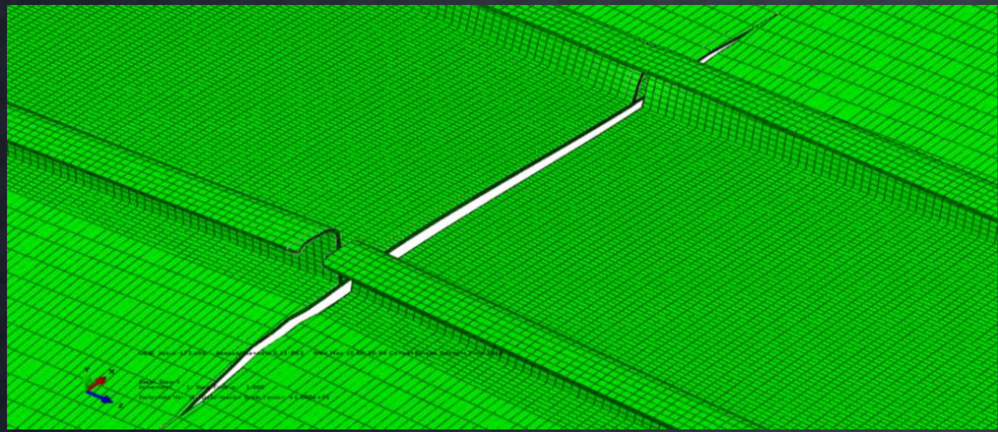
CS 3 – Numerical model of panel (XFEM)



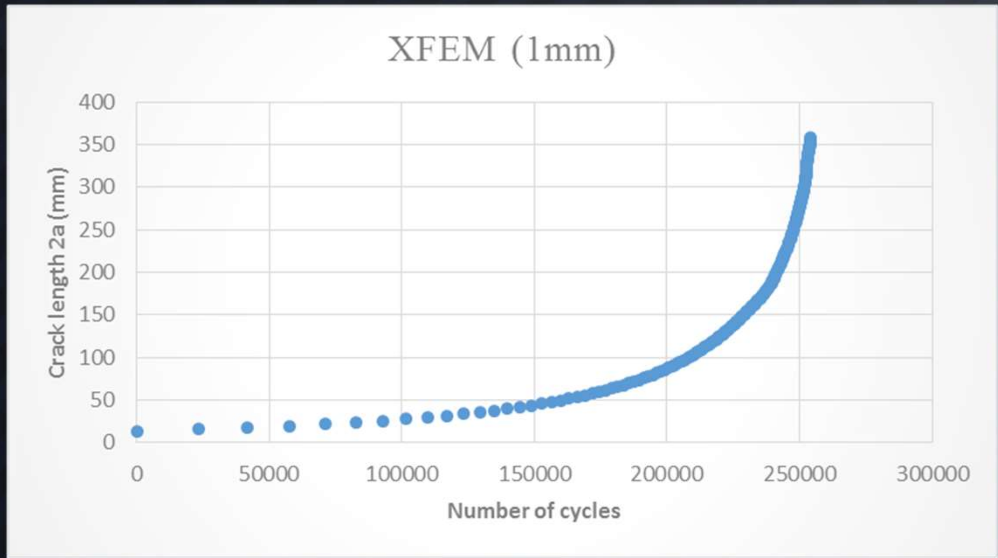


Model of 4-stringer plate (1mm mesh) with crack used in simulation

CS 3 – Numerical model of panel (XFEM)

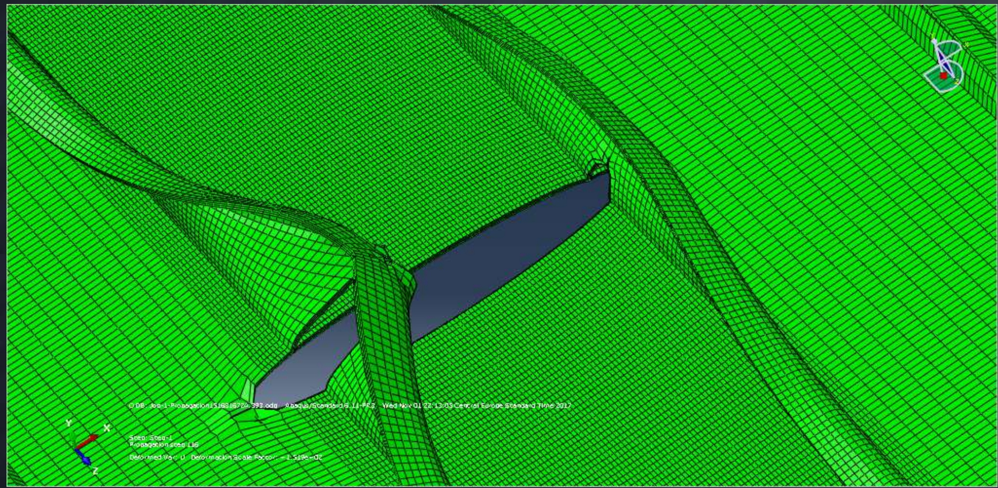


Crack after 160 steps of propagation

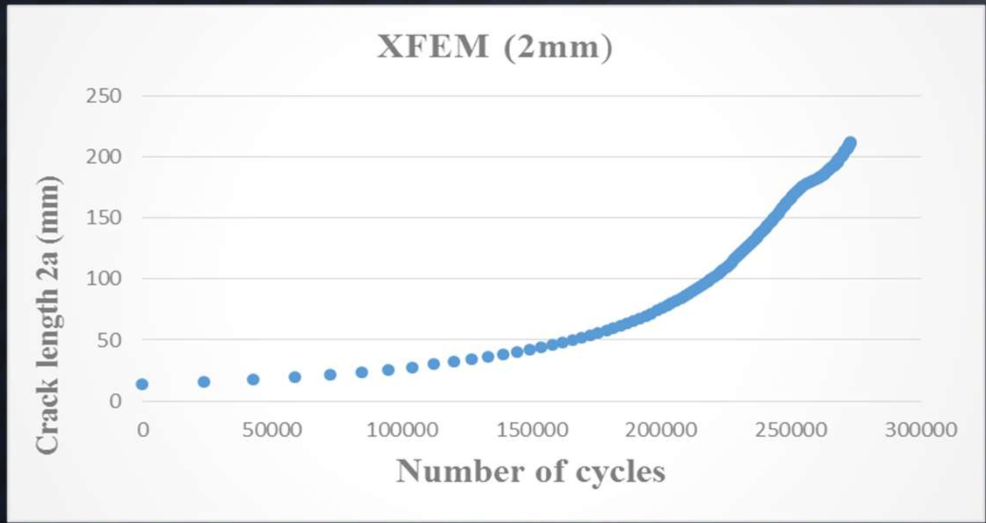


Crack growth vs. number of cycles (m = 3.174 and C = 1.77195x10^{-12} MPa mm$^{1/2}$)

CS 3 – Numerical model of panel (XFEM)

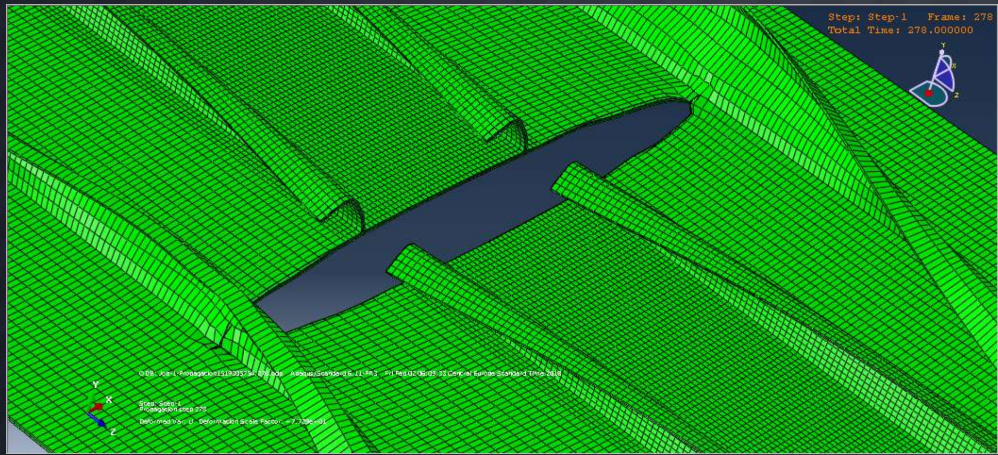


Model of 4-stringer plate (2mm mesh) with crack after 117 steps of propagation

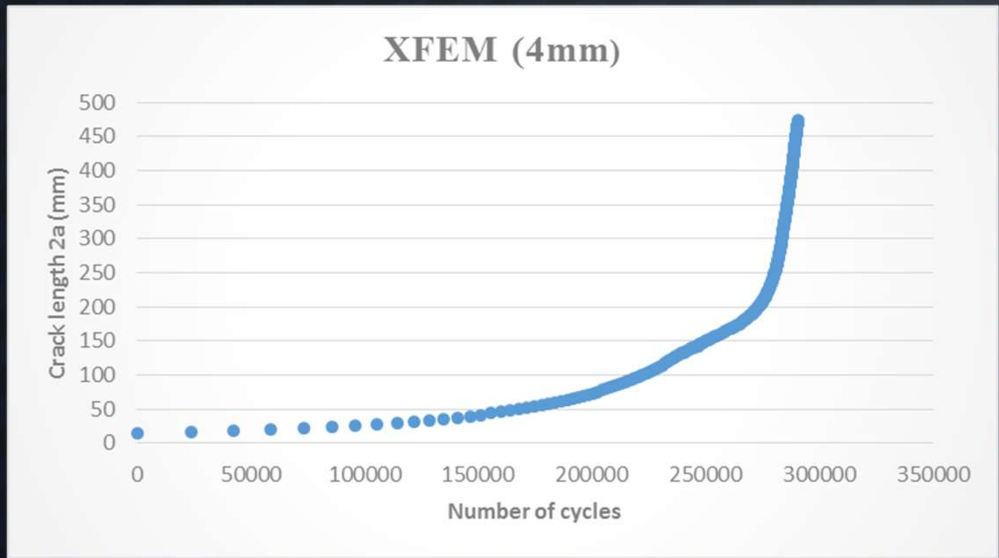


Crack growth vs. number of cycles (m = 3.174 and C = 1.77195×10^{-12} MPa $mm^{1/2}$)

CS 3 – Numerical model of panel (XFEM)

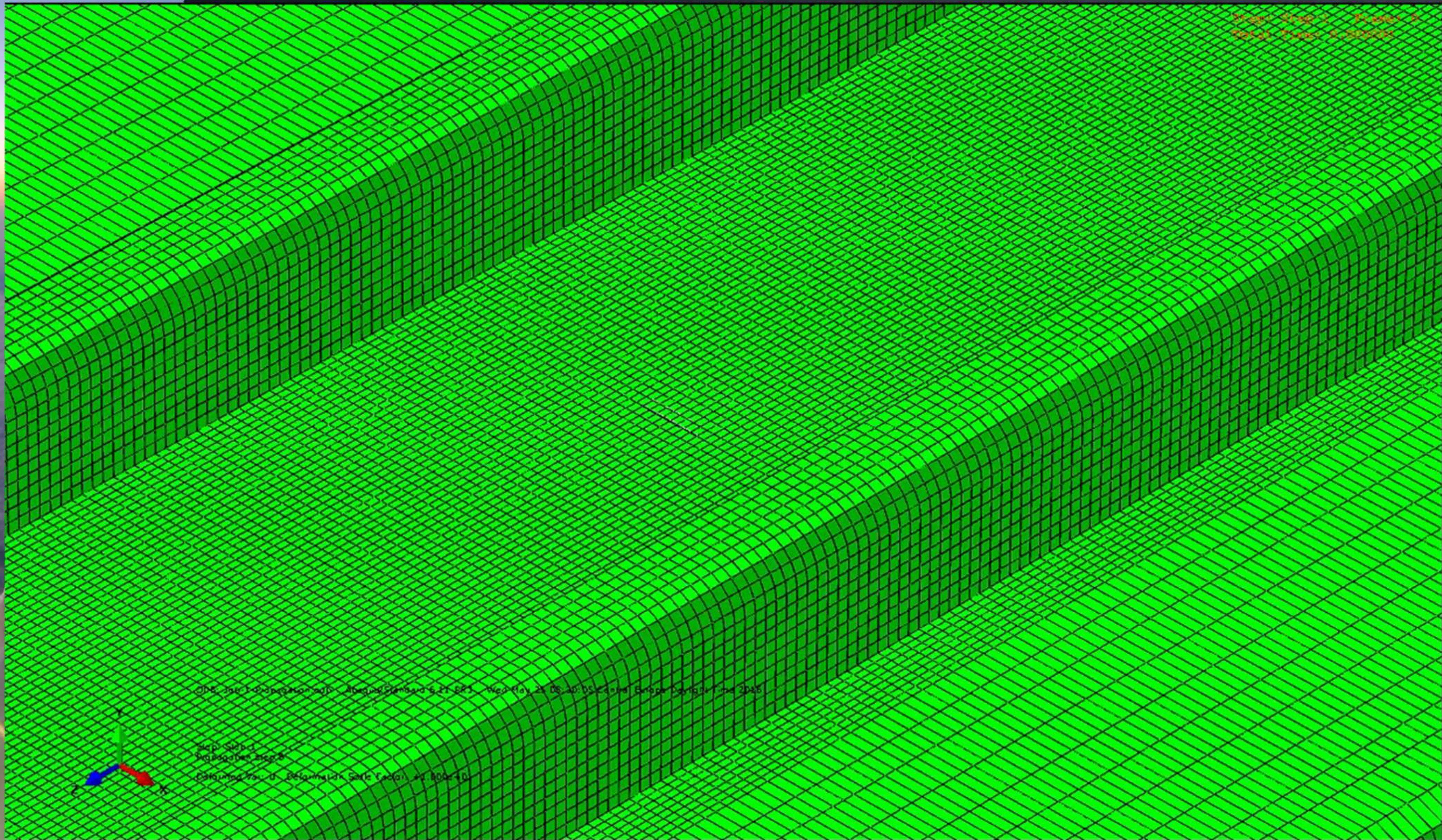


Model of 4-stringer plate (4mm mesh) with crack after 278 steps of propagation

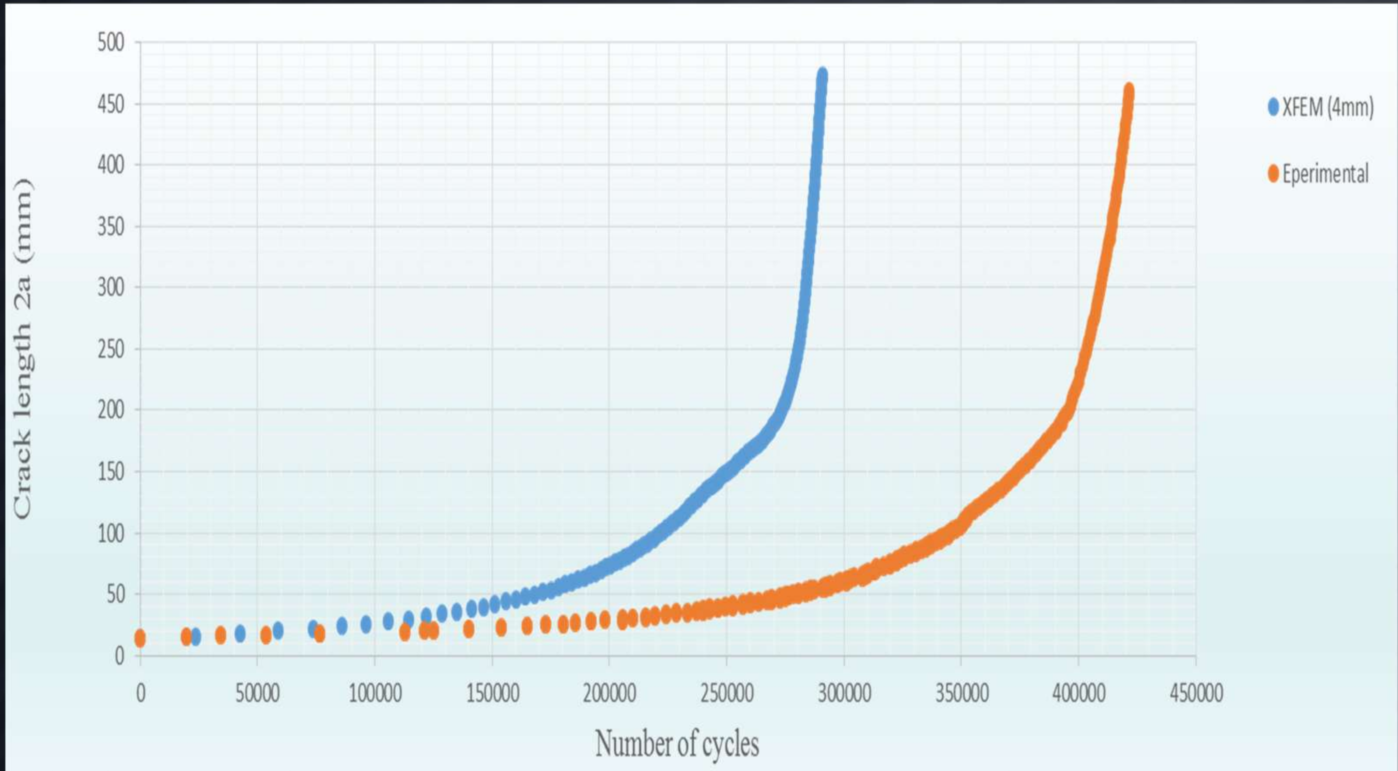


Crack growth vs. number of cycles (m = 3.174 and C = 1.77195x10^{-12} MPa mm$^{1/2}$)

CS 3 – Numerical model of panel (XFEM)

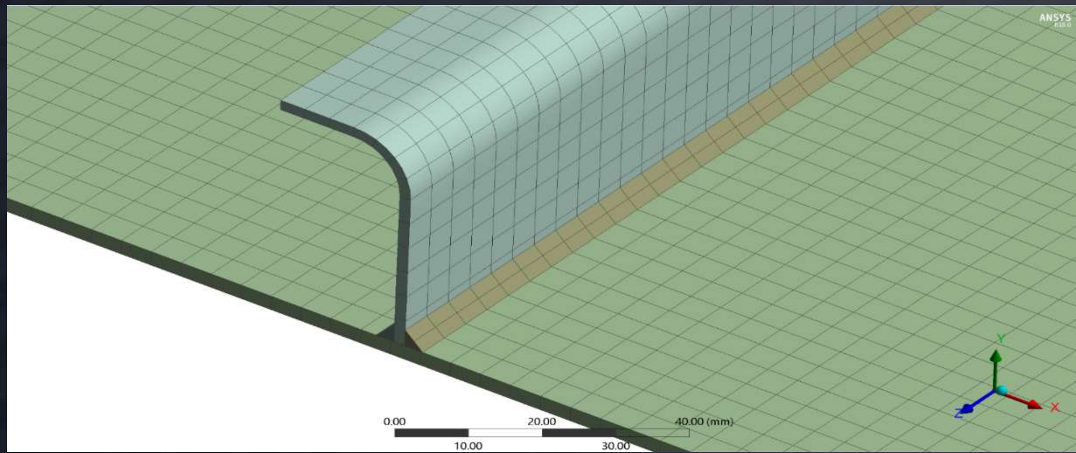


CS 3 – Comparison of values

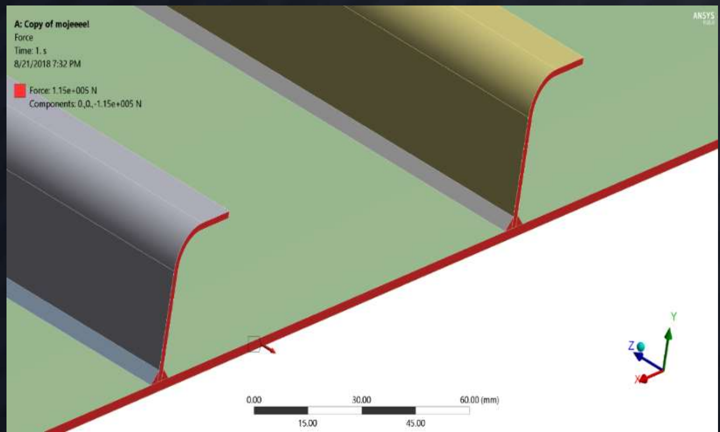


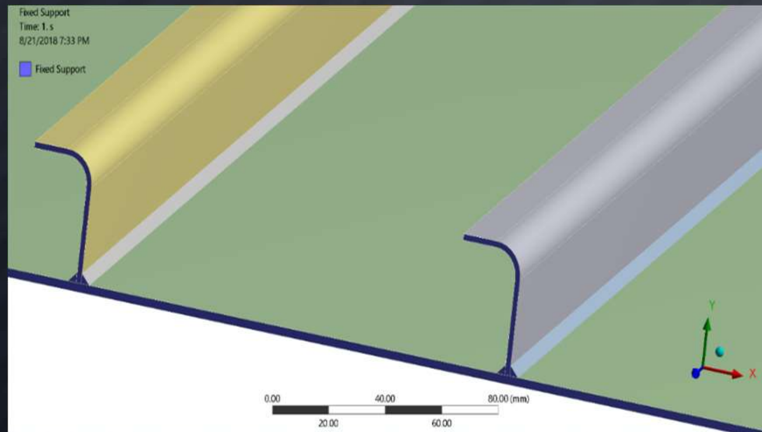
The number of cycles to critical crack length obtained in Abaqus is still less than the number of cycles obtained in the experiment (290743 cycles versus 422328 cycles; difference of about 31 %).

CS 3 – Improvements in numerical model



Mesh details of 4-stringer model. Weld line is presented.





Different definition of boundary conditions

Thank you for your attention!