



UNIVERSITÀ
DI PAVIA

1st Winter School on

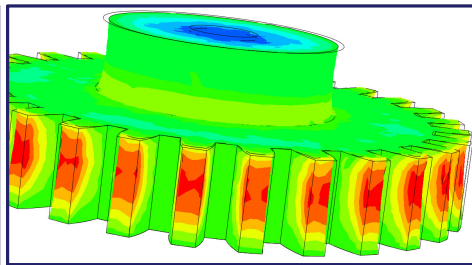
***Trends on Additive Manufacturing
for Engineering Applications***



Simulation for additive manufacturing: opportunities and challenges

Prof. Ferdinando Auricchio

Computational Mechanics and Advanced Material Group
University of Pavia



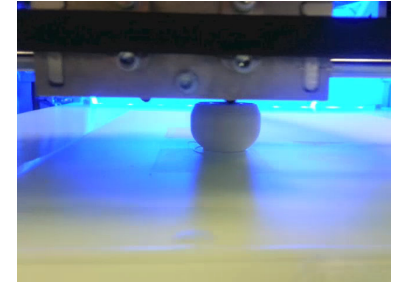
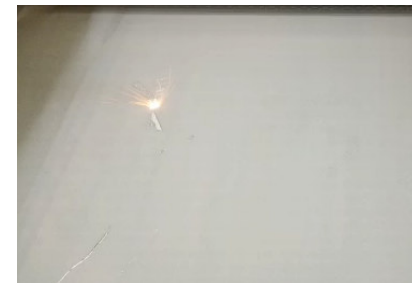
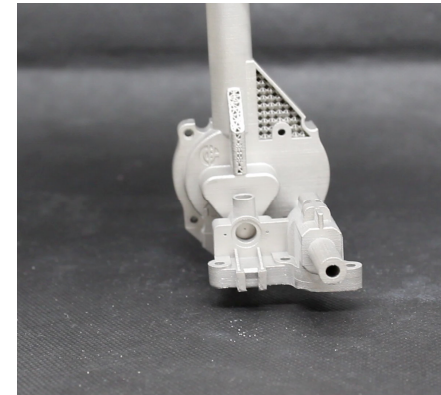
- **Introduction**
 - AM - 3DP: advantages, open problems
- **Design for additive**
 - Phase-field topology optimization: gradient material
 - Adaptive isogeometric analysis
 - Phase-field topology optimization: single material
- **Process simulations**
 - Immersed boundary approach
 - Melt pool: high fidelity simulations
 - Part-scale: low fidelity simulations
 - Two-level method
- **Product simulations**
 - Lattice components
 - Industrial components
- **Future activities & directions**
 - Innovative processes and materials
- **Conclusion**

Advantages

- Produce **complex geometries**: close to free-form flexibility
- Produce **single device made of multiple components** (assembling more parts into a single one)
- Combine different devices and geometries in a single printing batch
- **Green technology**: reduced waste
- **Accelerate design-testing-production** process chain (even in our labs)

Disadvantages

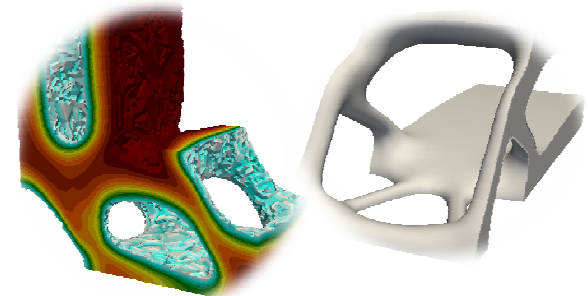
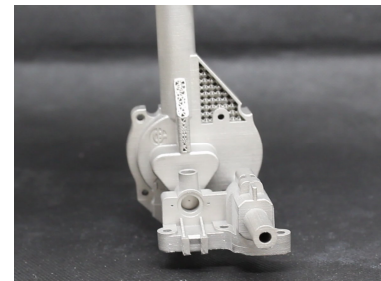
- Need of **support materials** (technology dependent)
- **Very localized physics** (multi-scale problem, technology dependent)
- Low speed (still a limitation)
- High cost (still a limitation)
- Interaction with further production steps (subtractive or finishing)



- **Introduction**
 - AM - 3DP: advantages, open problems
- **Design for additive**
 - Phase-field topology optimization: gradient material
 - Adaptive isogeometric analysis
 - Phase-field topology optimization: single material
- **Process simulations**
 - Immersed boundary approach
 - Melt pool: high fidelity simulations
 - Part-scale: low fidelity simulations
 - Two-level method
- **Product simulations**
 - Lattice components
 - Industrial components
- **Future activities & directions**
 - Innovative processes and materials
- **Conclusion**

Design for additive: challenges

- **Close-to-freeform** flexibility requiring **novel design approaches**
- **Topology and shape optimization** as tools for **design**, focusing on product functionality and production constraints



Topology optimization: goal

- Optimal distribution of given amount of material
- Minimize structure compliance (i.e., maximize stiffness)

Phase-field Method:

- No filtering methods required (cfr. SIMP approaches)

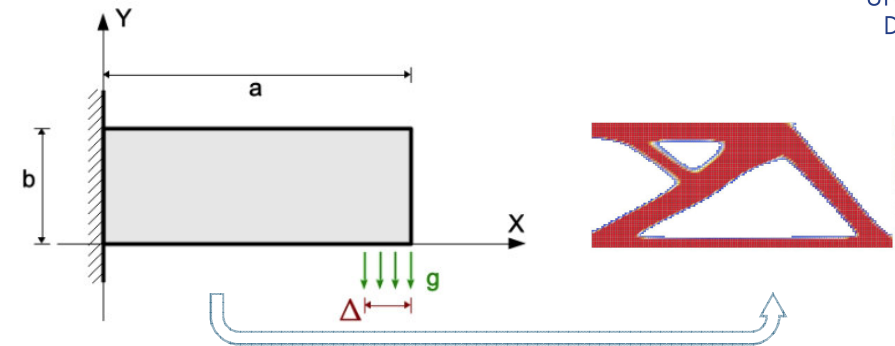
Limit discussion to for linear elastic problems

Introduce standard elastic problem in a domain Ω

$$\begin{aligned} \operatorname{div}[\mathbf{C} \boldsymbol{\varepsilon}(\mathbf{u})] &= \mathbf{b} & \text{in } \Omega \\ \mathbf{u} &= \mathbf{0} & \text{on } \Gamma_D \\ [\mathbf{C} \boldsymbol{\varepsilon}(\mathbf{u})] &= \mathbf{t} & \text{on } \Gamma_N \end{aligned}$$

Introduce description of meso-structure (variable density, lattice):

- Obtain a graded design, i.e., structure with varying density



Objective

Minimize structure compliance:

$$\int_{\Omega} \mathbf{b} \cdot \mathbf{u} \, d\Omega + \int_{\Gamma_N} \mathbf{t} \cdot \mathbf{u} \, d\Gamma$$

properly distributing material in Ω

Acknowledgments: M.Carraturo, E.Rocca, A.Reali (UniPV & IMATI-CNR), E.Bonetti (Università di Milano & IMATI-CNR), D.Hömborg (WIAS Institute Berlin)

Publications:

- Carraturo, Rocca, Bonetti, Hömborg, Reali, FA. *Graded-material design based on phase-field and topology optimization*. Computational Mechanics, Vol. 64, 1589–1600 (2019)
- FA, Bonetti, Carraturo, Hömborg, Reali, Rocca. *A phase-field based graded-material topology optimization with stress constraint*. M3AS, Vol. 30 (08), 1461–1483 (2020)

- Phase-field variables

φ : material presence

$$0 \leq \varphi \leq 1$$

[$\varphi = 0$: no material, $\varphi = 1$: material]

χ : material features (scalar quantity=density)

$$0 \leq \chi \leq \varphi$$

[$\chi = 0$: material B, $\chi = \varphi$: material A]

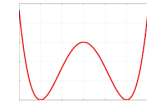
- Material elasticity continuously varies from a soft (\mathbf{C}_B) to a stiff material (\mathbf{C}_A)

$$\mathbf{C} = \mathbf{C}_B \chi + \mathbf{C}_A (1 - \chi)$$

- Classical topology optimization: minimize compliance, complemented with a perimeter measure

$$\mathcal{J}(\phi, \mathbf{u}) = \int_{\Omega} \phi \mathbf{b} \cdot \mathbf{u}(\phi) d\Omega + \int_{\Gamma_N} \mathbf{t} \cdot \mathbf{u}(\phi) d\Gamma + \kappa_{\phi} \int_{\Omega} \left[\gamma_{\phi} \|\nabla \phi\|^2 + \frac{1}{\gamma_{\phi}} \psi_0(\phi) \right] d\Omega$$

Double-well potential



$$\psi_0(\phi) = (\phi - \phi^2)^2$$

- New topology optimization: Introduce extra-penalization for the gradient of second scalar field χ

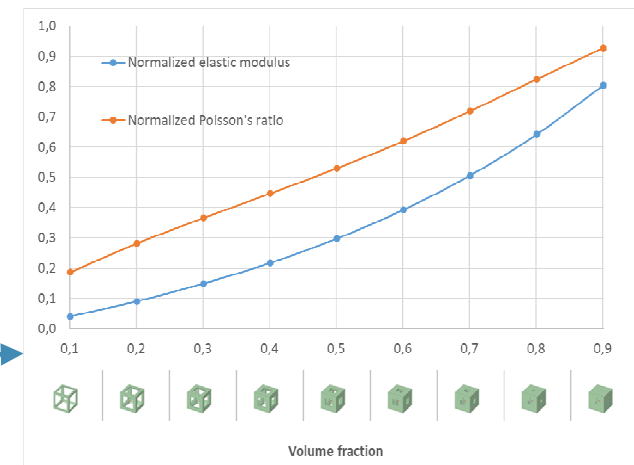
$$\mathcal{J}^M(\phi, \mathbf{u}, \chi) = \mathcal{J} + \kappa_{\chi} \int_{\Omega} \gamma_{\chi} \|\nabla \chi\|^2 d\Omega$$

- Introduce volume constraint & complementary problem (equilibrium)

$$\mathcal{L}^M(\phi, \mathbf{u}, \chi) = \mathcal{J}^M + \lambda \left[\int_{\Omega} (\phi - \bar{v}) d\Omega \right] + \mathcal{S}^M$$

$$\mathcal{S}^M(\phi, \mathbf{u}, \chi, \mathbf{p}) = \int_{\Omega} \boldsymbol{\varepsilon}(\mathbf{u}) : \mathbf{C}(\phi, \chi) \boldsymbol{\varepsilon}(\mathbf{p}) d\Omega - \int_{\Gamma_N} \mathbf{g} \cdot \mathbf{p} d\Gamma$$

Elastic tensors also from homogenization on a lattice RVE with periodic BCs



- Minimization process

- Allen-Cahn gradient flow, i.e. steepest descent pseudo-time stepping method, with time-step increment τ

$$\frac{\gamma_\phi}{\tau} \int_{\Omega} (\phi_{n+1} - \phi_n) v_\phi d\Omega + \kappa_\phi \gamma_\phi \int_{\Omega} \nabla \phi \cdot \nabla v_\phi d\Omega + \dots = 0$$

$$\frac{\gamma_\chi}{\tau} \int_{\Omega} (\chi_{n+1} - \chi_n) v_\chi d\Omega + \kappa_\chi \gamma_\chi \int_{\Omega} \nabla \chi \cdot \nabla v_\chi d\Omega + \dots = 0$$

- Alternate solution of gradient flow and equilibrium problem

- Finite element approximation of fields

- Discretize domain using quads
- Piecewise linear basis functions (except that for global fields)

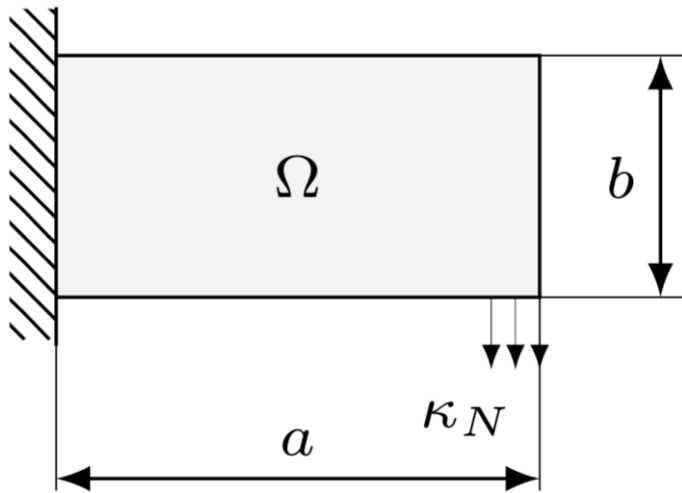
- Adopt a two-step algorithm

- solve equilibrium to get displacement vector
- solve Allen-Cahn gradient flow to get phase-field, material field, Lagrange multiplier vectors
- rescale to fulfilling constraints

```

input : Q, Q_φ, Q_χ, Q_λ, φ_0, χ_0
output: Optimal topology
1 φ_n ← φ_0
2 χ_n ← χ_0
3 while (Δ_φ ≥ tol or Δ_χ ≥ tol) and n ≤ max_iter do
4   ũ_{n+1} ← solve(26)
5   (φ̃_{n+1}^*, χ̃_{n+1}^*, λ̃_{n+1}) ← solve(27)
6   φ̃_{n+1} ← rescale (φ̃_{n+1}^*) to [0, 1]
7   χ̃_{n+1} ← rescale (χ̃_{n+1}^*) to [0, φ]
8   update(Δ_φ and Δ_χ)
9   φ_n ← φ_{n+1}
10  χ_n ← χ_{n+1}
11 end
    
```

- Use $\Delta\phi$ L2-norm increments as convergence criteria



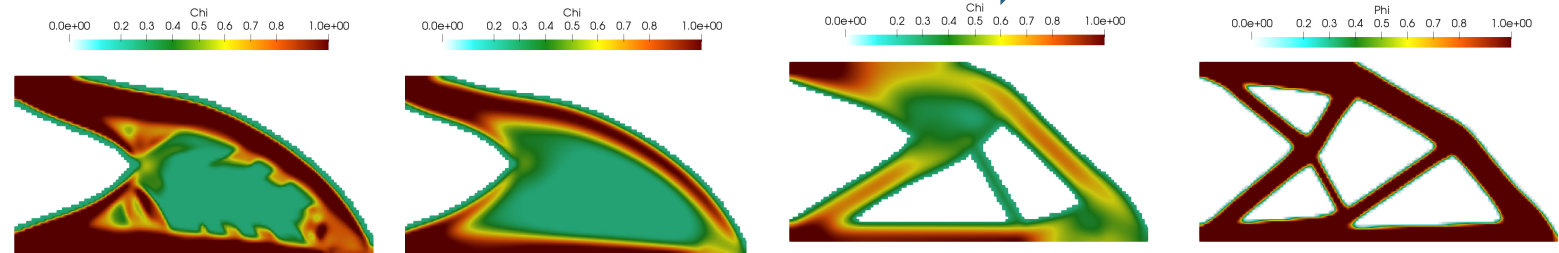
GradsDensity_Function



Grey = no material
Color (different than grey) = material
Different color = different materials

Sensitivity study for γ_χ

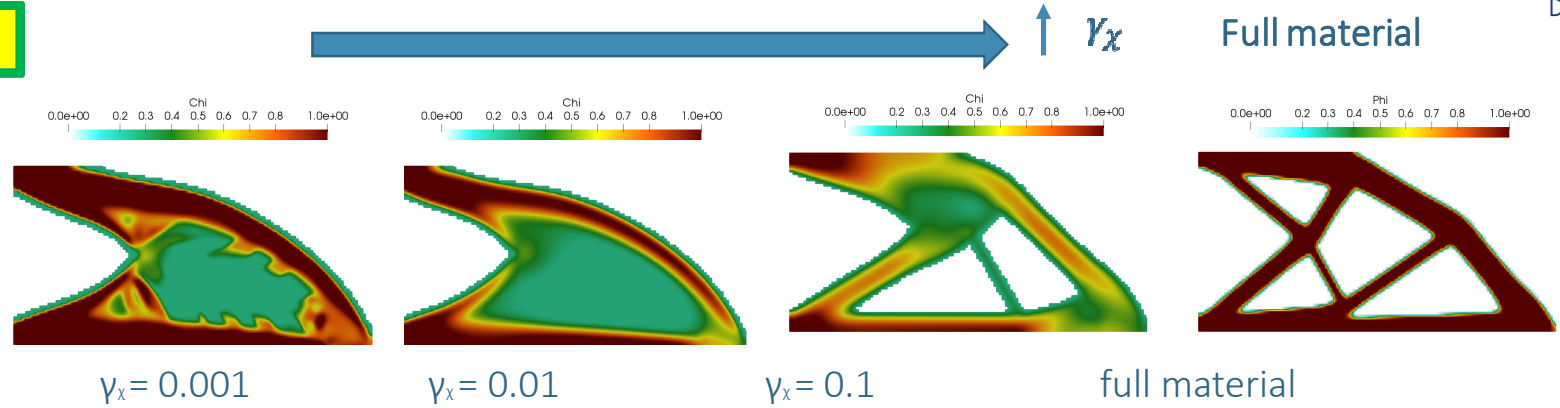
Increasing penalty gradient material, structures closer to *black-and-white* solution



- Multi-material distribution very different from single-material topology (right figure) for small values of γ_χ
- Voids present in single-material structure replaced by areas of soft material in multi-material structure
- When thickness of diffuse interface is too small compared to element size, solution does not converge

Sensitivity study for γ_x

Grey = no material
Color = material
Different color = different materials



- Volume fraction index m_ϕ : to estimate material volume
- Material fraction index m_x : to estimate material amount (density)

$$m_\phi = \frac{1}{\Omega} \int_{\Omega} \phi d\Omega$$

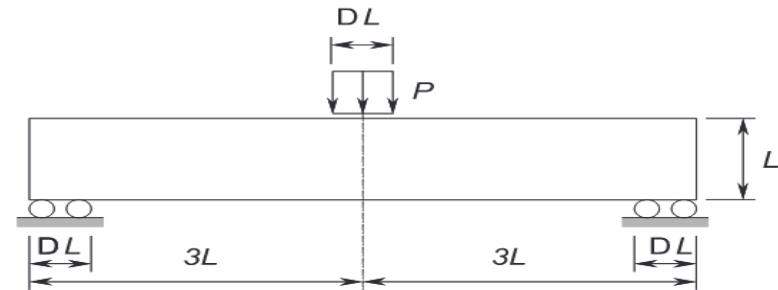
$$m_x = \frac{1}{\Omega} \int_{\Omega} \chi d\Omega$$

γ_x	Compliance	m_x	Converge?
0.001	105.3	0.380	NO
0.005	122.9	0.265	YES
0.01	133.0	0.245	YES
0.02	141.9	0.230	YES
0.05	154.0	0.225	YES
0.1	165.4	0.201	YES

Employing a softer material decreases body compliance, leading to heavier structures compared to the homogeneous material case

- Messerschmitt-Bölkow-Blohm (MBB) beam

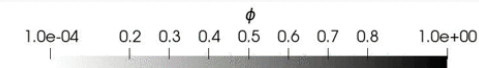
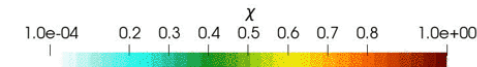
- Applied force = 25 N
- Material: RGD851 rigid polymer from Stratasys (E=2.3 GPa and $\nu=0.3$)
- 3D printer machine: Stratasys Objet 260 Connex 3
- Volume fraction = 0.6
- Mass fraction = 0.4



Messerschmitt-Bölkow-Blohm GmbH; Payten et al. 1998; Bulman et al. 2001

- Results

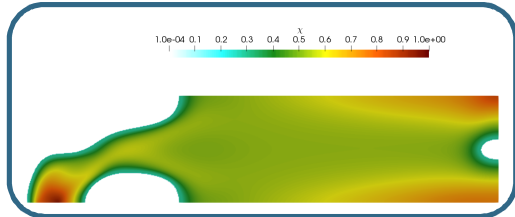
1. Black-and-white structure indicates material presence
2. Density continuously re-distributed within material region



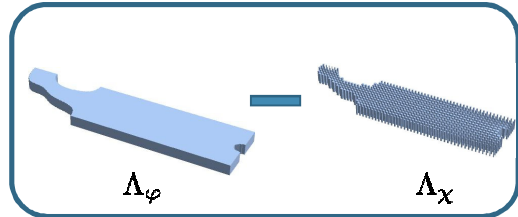
Acknowledgments: G. Alaimo, M. Carraturo, E. Rocca, A. Reali (UniPV & IMATI-CNR)

Publications: Alaimo, Carraturo, Rocca, Reali, FA. *Functionally graded material design for plane stress structures using phase field method*, II International Conference on Simulation for Additive Manufacturing - Sim-AM 2019

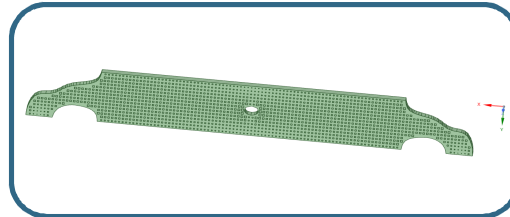
Objective: evaluate optimized versus uniform (same weight) specimen in terms of max. displacements



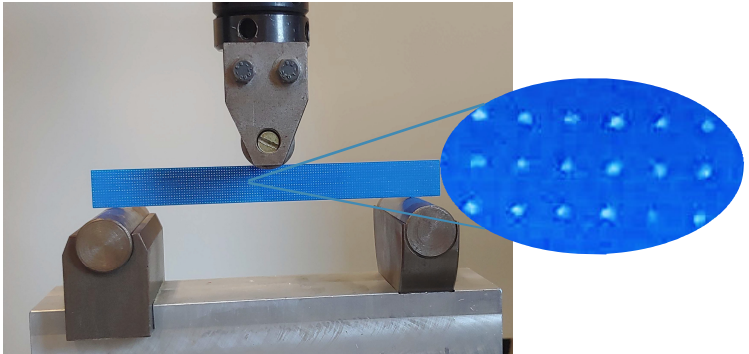
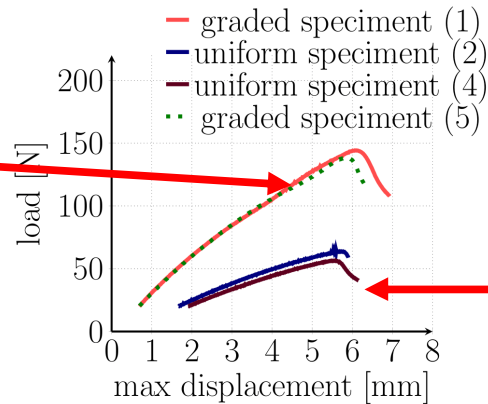
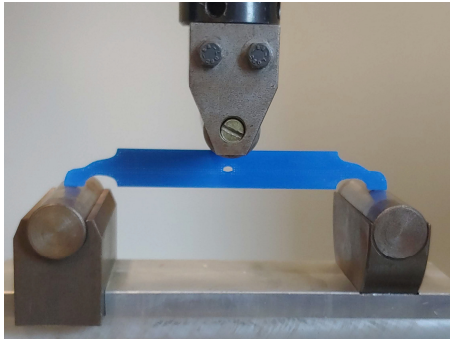
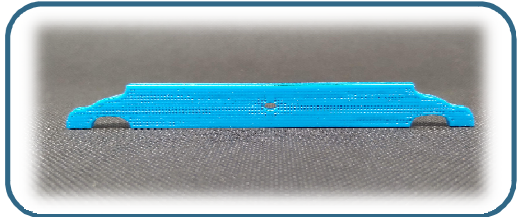
Discrete map of field variables



Generate 3D virtual model



3D printing



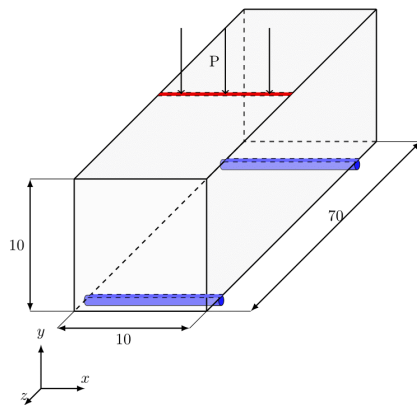
Results: for the same load, we observe a **reduction of 50%** as max. displacement

Special thanks to: G.Alaimo (ProtoLab) & S.Marconi (3D4Med)

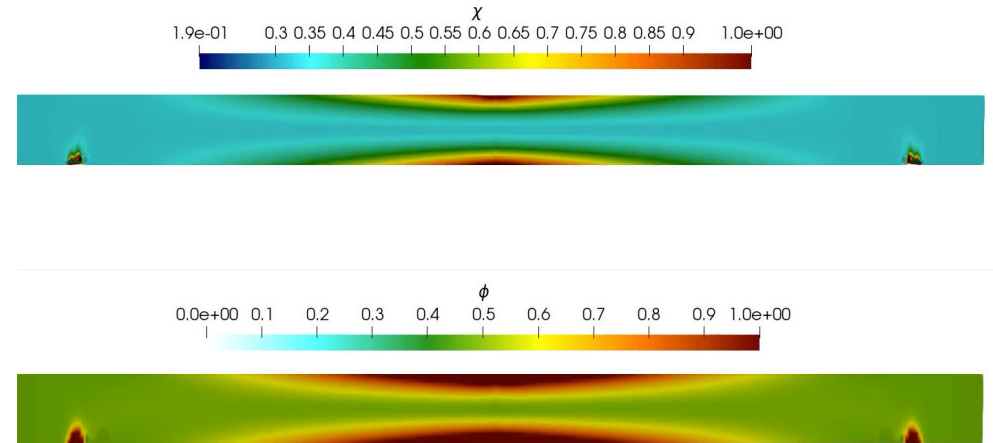
Acknowledgments: G.Alaimo, M.Carraturo, E.Rocca, A.Reali (UniPV & IMATI-CNR)

Publications: Alaimo, Carraturo, Rocca, Reali, FA *Functionally graded material design for plane stress structures using phase field method*, II Int.Conf. Simulation for AM - Sim-AM 2019

- **Objective:** extend 2D workflow to 3D structures to evaluate the performance of the optimization for a 3D case
- **Example:** MBB-beam problem

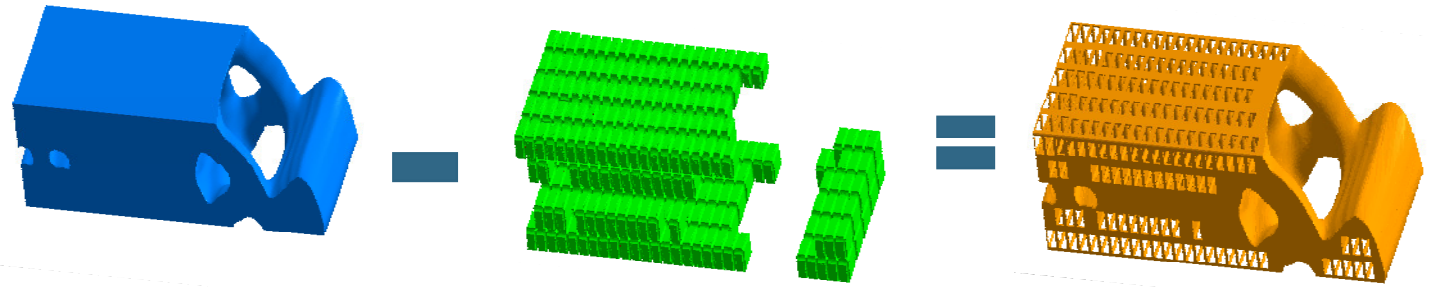


Optimize structure using phase-field gradient top-opt



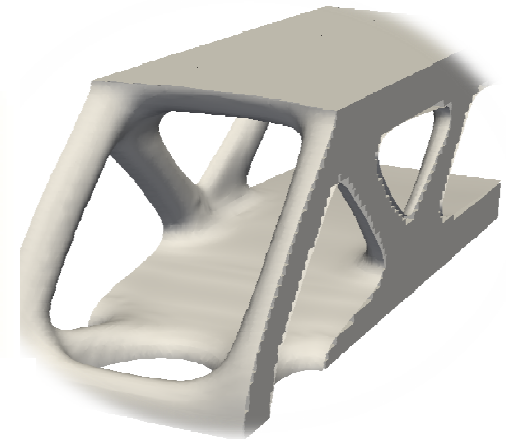
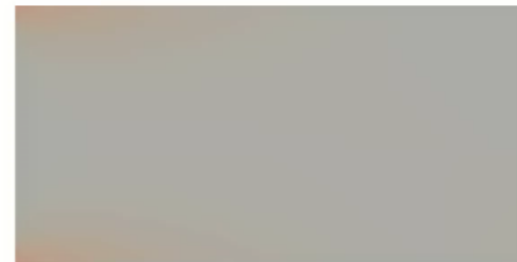
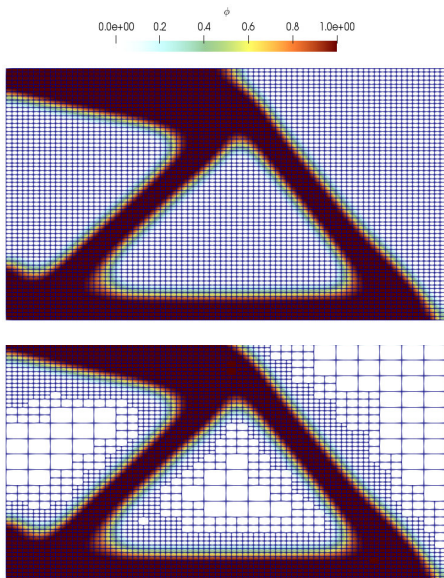
Slicing & 3D model reconstruction

On-going work...
3D printing & Experiment



- **Idea:** Approximate mechanical and phase-field solution space using IGA, since higher continuity of IGA basis functions very effective for phase-field methods
- Adaptive Isogeometric Analysis as presented in **Henning et al. 2016** allows to **locally concentrate the computational effort** at the material interface **without any loss of accuracy**
- Single material

Work in progres...



- 60% reduction in terms of DOFs
- 40% less CPU time
- Higher improvement are likely expected for the 3D case...



Acknowledgments: Markus Kästner, Paul Henning, Leonhard Heindel (TU Dresden), M.Carraturo, A.Reali (UniPV & IMATI-CNR)

Publications: Henning, Heindel, Carraturo, Reali, FA, Kästner. *Projection Methods in Adaptive Isogeometric Analysis and its Application to Topology Optimization*, Proceedings in Applied Mathematics and Mechanics (*accepted*).

Single material, hence single scalar field ϕ

Volume-constrained formulation:

- Formulation including compatibility, constitutive, equilibrium equation into a single functional

$$\mathcal{L}^{vc}(\phi, \mathbf{u}, \boldsymbol{\varepsilon}, \boldsymbol{\sigma}) = \underbrace{J(\phi, \mathbf{u})}_{\phi \text{ bounding}} + \underbrace{\kappa_b \mathcal{B}(\phi)}_{\text{Elastic energy}} - \underbrace{\mathcal{E}^{el}(\boldsymbol{\varepsilon}, \phi)}_{\text{Hu-washizu}} + \underbrace{\int_{\Omega} \boldsymbol{\sigma} : (\boldsymbol{\varepsilon} - \nabla^s \mathbf{u}) d\Omega}_{\text{Volume constraint}} + \lambda \left[\int_{\Omega} (\phi - \bar{v}) d\Omega \right]$$

With

$$\mathcal{B}(\phi) = \int_{\Omega} b(\phi) d\Omega \quad \text{with} \quad b(\phi) = \begin{cases} (\phi - 1)^2/2 & \text{if } \phi > 1 \\ 0 & \text{if } 0 \leq \phi \leq 1 \\ \phi^2/2 & \text{if } \phi < 0 \end{cases} \quad \mathcal{E}^{el}(\boldsymbol{\varepsilon}, \phi) = 1/2 \int_{\Omega} \mathbf{C}(\phi) \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon} d\Omega$$

- No need of re-normalizing volume fraction
- All eqns into a single functional, i.e. possible monolithic solution schemes

Volume-minimization formulation:

Not imposing a constraint on volume but looking for volume minimization

$$\mathcal{L}^{vm}(\phi, \mathbf{u}, \boldsymbol{\varepsilon}, \boldsymbol{\sigma}) = J(\phi, \mathbf{u}) + \kappa_b \mathcal{B}(\phi) - \mathcal{E}^{el}(\boldsymbol{\varepsilon}, \phi) + \int_{\Omega} \boldsymbol{\sigma} : (\boldsymbol{\varepsilon} - \nabla^s \mathbf{u}) d\Omega + \frac{\kappa_v}{2} \left[\int_{\Omega} \phi^2 d\Omega \right]$$

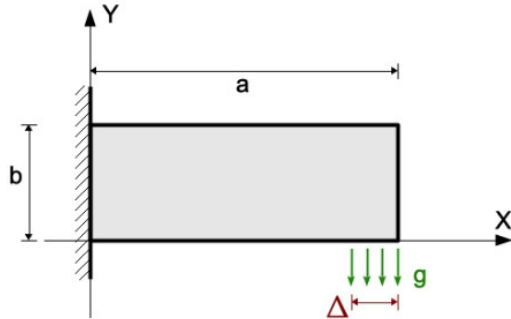
Results

- Relation between \mathcal{L}^{vc} and \mathcal{L}^{vm} : possible to show that they return the same solution
- Second approach is not a saddle point problem but a true minimization (improved convergence properties)

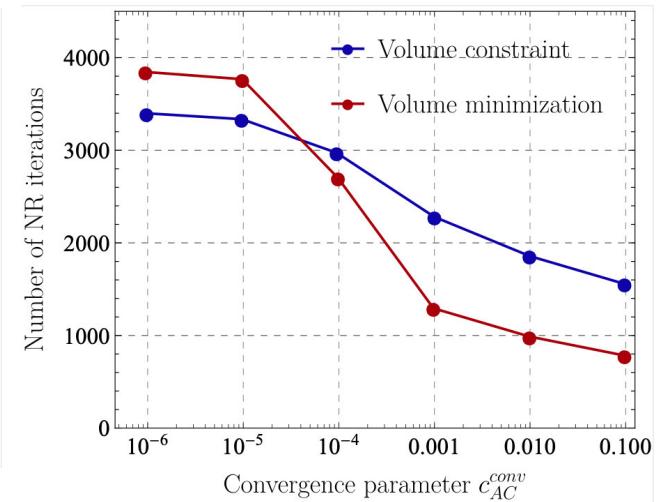
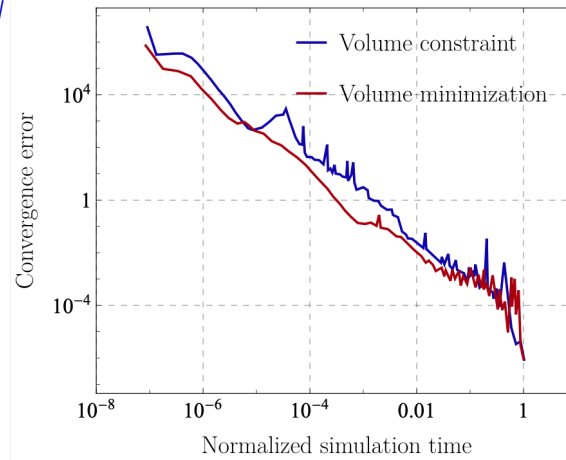
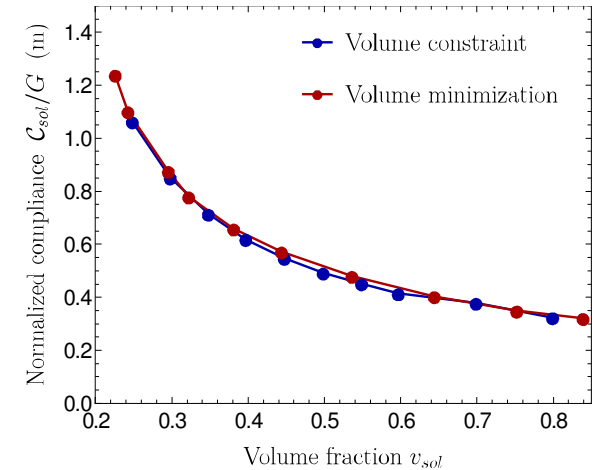
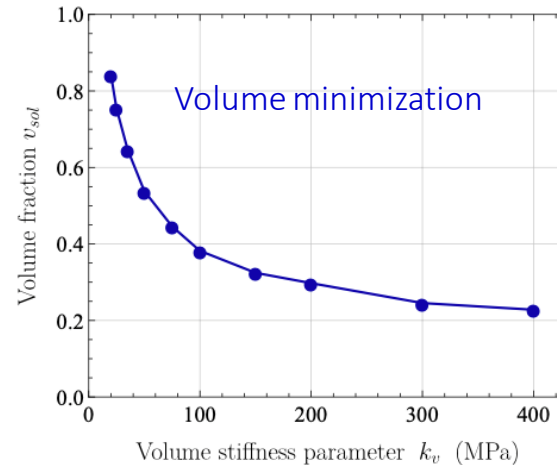
Acknowledgments: E.Rocca, A.Reali (UniPV & IMATI-CNR), U.Stefanelli (University of Vienna), M.Marino (University of Roma Tor Vergata)

Publications: Marino, FA, Reali, Rocca, Stefanelli (2020), *Mixed variational formulations for structural topology optimization based on phase-field*, submitted

Accurate and extensive numerical investigation



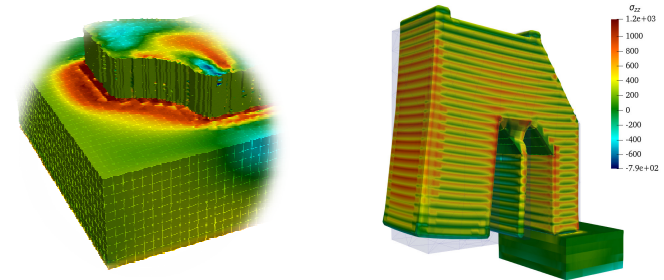
- Increasing volume stiffness reduces volume fraction
- Solutions from two formulations are identical in terms of compliance C_{sol} and interface perimeter P_{sol} for the same value of final volume fraction
- Volume minimization shows improved convergence
 - error decreases more monotonically (!)
- Volume minimization converges faster in terms of
 - simulation time
 - number of Newton iterations



- **Introduction**
 - AM - 3DP: technologies, materials, advantages, open problems
- **Design for additive**
 - Phase-field topology optimization: gradient material
 - Adaptive isogeometric analysis
 - Phase-field topology optimization: single material
- **Process simulations**
 - Immersed boundary approach
 - Melt pool: high fidelity simulations
 - Part-scale: low fidelity simulations
 - Two-level method
- **Product simulations**
 - Lattice components
 - Industrial components
- **Future activities & directions**
 - Innovative processes and materials
- **Conclusion**

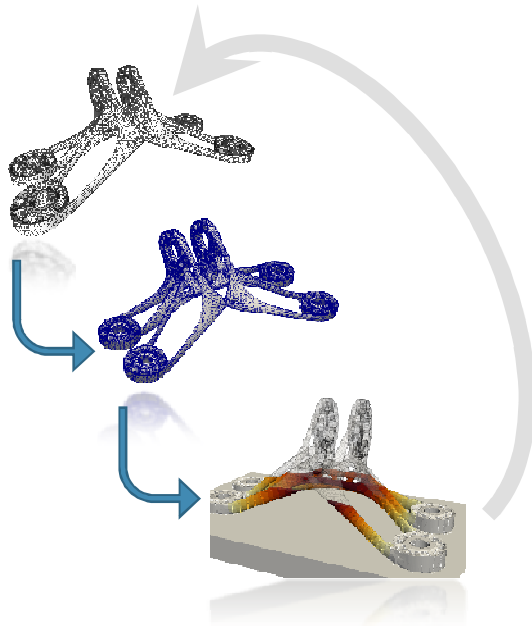
Process simulations: challenges

- **Large scale range** both in space and time
- **Complex physical phenomena** to be modeled
- Predict defects due to process



Focus on the most industrially relevant technology: laser powder bed fusion for metal components (LPBF)

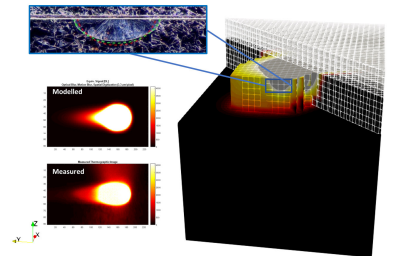
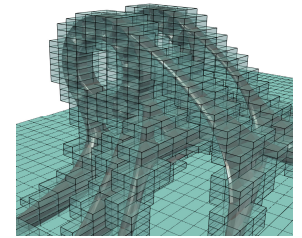
Standard AM-design process



- 3D virtual model is developed within a CAD environment
- Geometry to be repaired
- Conform mesh generated
- Finite element analysis of the process
- To update the geometry, need to go back to CAD software and start procedure once again ...

AM-design-through-analysis

- Thermo-mechanical analyses can be performed directly on CAD models
- STL repair step required only once the final design ready to be printed
- Remarkable computational speed-up for multi-layer high-fidelity analyses of complex geometrical features



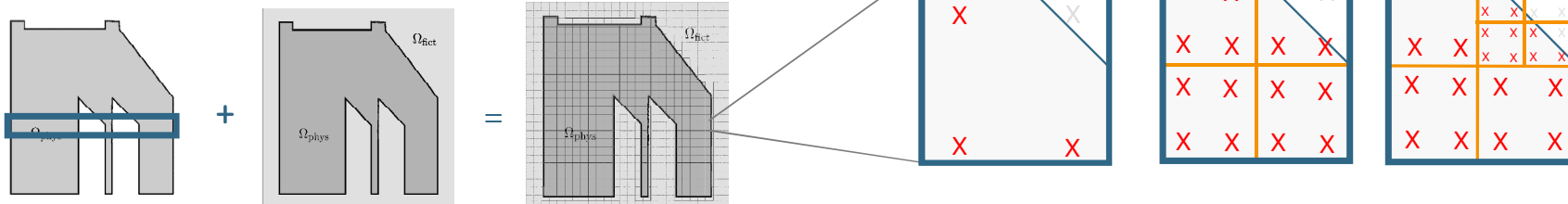
Acknowledgments: Ernst Rank, Stefan Kollmannsberger, John Jomo, Ali Özcan, Nils Zander (TUM), M.Carraturo, A.Reali (UniPV & IMATI-CNR)

Publications:

- Kollmannsberger, Özcan, Carraturo, Zander, Rank. *A hierarchical computational model for moving thermal loads and phase changes with applications to selective laser melting*. CAMWA, Vol. 75 (5), 1483-1497 (2018)
- Carraturo, Jomo, Kollmannsberger, Reali, FA, Rank. *Modeling and experimental validation of an immersed thermo-mechanical part-scale analysis for laser powder bed fusion processes*. Additive Manufacturing, Vol. 36, 101498 (2020)

The Finite Cell Method (FCM)

Initial domain discretization



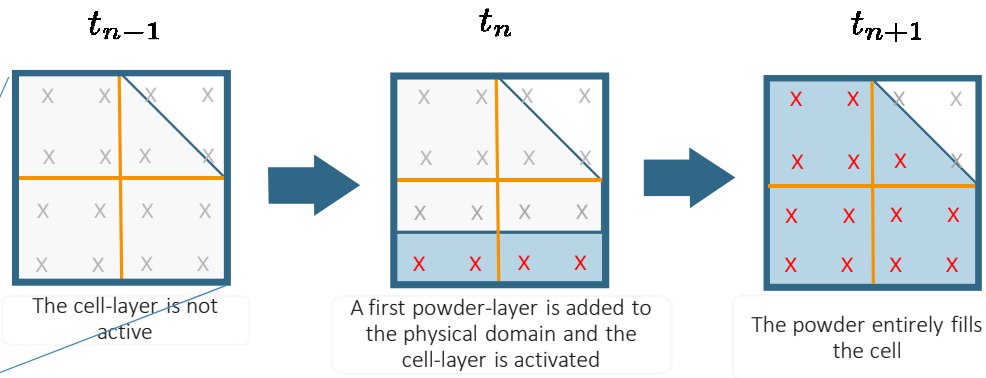
- Weak form modified using a parameter α evaluated at Gauss points
- Integration points distributed on sub-cells to accurately integrate over discontinuities at boundaries

$$\alpha(\mathbf{x}) a(\mathbf{u}, \mathbf{v}) = l(\mathbf{v})$$

$$\text{with } \alpha(\mathbf{x}) = \begin{cases} 1 & \forall \mathbf{x} \in \Omega_{phys} \\ 0 & \forall \mathbf{x} \notin \Omega_{phys} \end{cases}$$

Application to growing domains

- LPBF is a **layer-by-layer** process
- Physical domain **grows** during the process
- Distinguish among **cell-layers** (where shape functions are defined) and **powder-layers** (where Gauss points are activated)



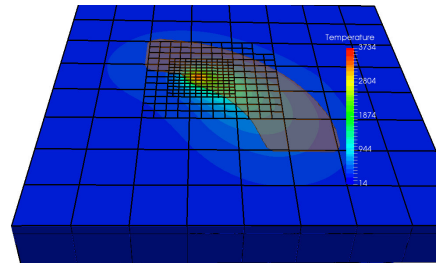
Immersed boundary approach for growing domain: different scale approaches

- Due to problem complexity, need to choose a-priori solution scale
- Choose quantities of interested



Input parameters	Range values
Laser power	100÷1000 [W]
Laser speed	0.2÷1.5 [m/s]
Laser spot radius	25÷100 [μm]

Melt-pool analysis High-fidelity simulation



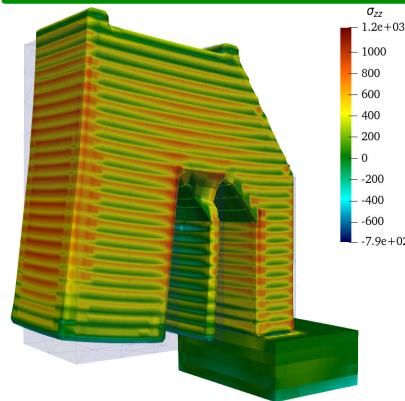
Objective

- Predict **temperature and stress state** at the **melt-pool length-scale** (element size ~ 10μm)
- Evaluate melt-pool shape and cooling rate

Model features

- **Few laser strokes** can be simulated (10÷100 mm length)
- Powder is included in the model
- Phase-change has to be taken into account

Part-scale analysis Low-fidelity simulation

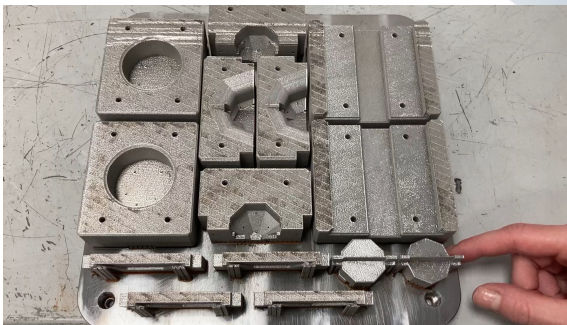


Objective

- Predict **part deflection** after base plate removal
- Evaluate **residual stresses** in the final component

Model features

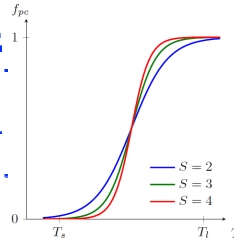
- **Complete process** is simulated (including post-processing steps, e.g. part removal)
- Powder modeled as conduction BC, not included in the domain
- Latent heat usually neglected



- Heat transfer equation

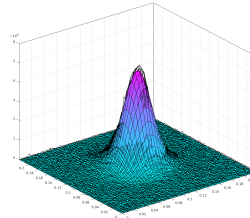
$$\rho c \dot{T} + \rho L \dot{f}_{pc} - \nabla(k \nabla T) = 0 \quad \text{in } \Omega$$

$$f_{pc} = \frac{1}{2} \left[\tanh \left(S \frac{2}{T_l - T_s} \left(T - \frac{T_s + T_l}{2} \right) \right) + 1 \right]$$



- Initial conditions

$$T(\mathbf{x}, t) = T_0 \quad \text{at } t = 0$$



- Boundary conditions

$$k \nabla T(\mathbf{x}, t) \cdot \mathbf{n} = q^s + q^L \quad \text{on } \Gamma_N$$

Radiation heat flux: $q^s = \sigma \epsilon (T^2 + T_e^2)(T_e^2 - T^2)$

Laser heat source: $q^L = \frac{2Q\eta}{\pi r^2} \exp \left[-2 \left(\frac{y - y_0}{r^2} + \frac{x - x_0}{r^2} \right) \right]$

Obtained fitting measured data with a gaussian distribution

- Mechanical equation

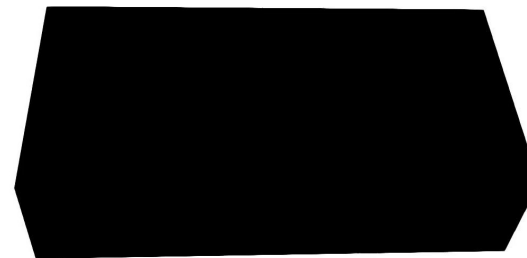
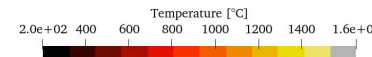
$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = \mathbf{0}$$

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^{th} + \boldsymbol{\epsilon}^{el} + \boldsymbol{\epsilon}^{pl}$$

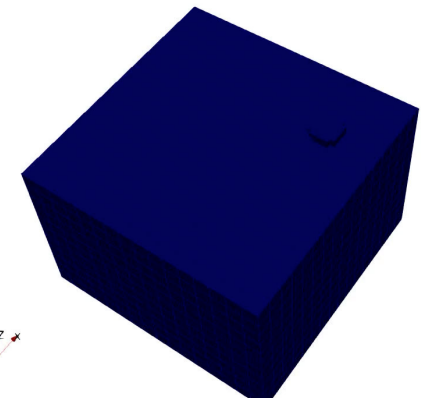
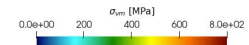
$$\boldsymbol{\epsilon}^{th} = \alpha^{th} \Delta T \mathbf{I}$$

$$\boldsymbol{\epsilon}^{pl} = \dot{\gamma} \frac{\partial \Phi}{\partial \boldsymbol{\sigma}}$$

$$\Phi = \sigma_{vm} - \sigma_y(\gamma, T) \leq 0$$



Thermal problem



Mechanical problem