

1st Winter School on

# Trends on Additive Manufacturing for Engineering Applications

24-28 January 2021



## The process of fracture in soft materials



Michele Terzano, PhD

*Department of Engineering & Architecture, University of Parma*

# Soft materials

What is soft matter?

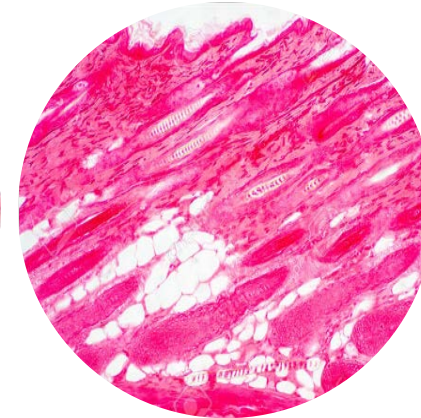
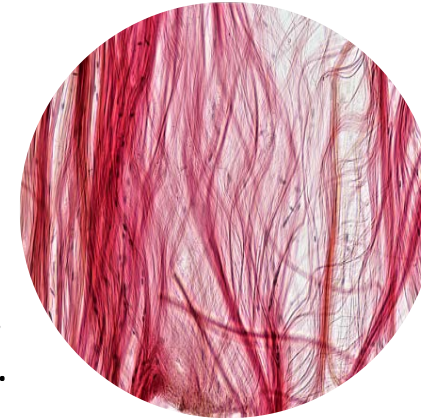


## Soft polymers

elastomers, colloids,  
liquid-crystals polymers,  
hydrogels, foams ...

## Soft tissues

skin, muscles, tendons,  
blood vessels, organs ...



Soft materials have ...

low initial elastic modulus

high ultimate tensile strain

nonlinear stress-strain relationship

time dependent behaviour

and, in addition, soft tissues show ...

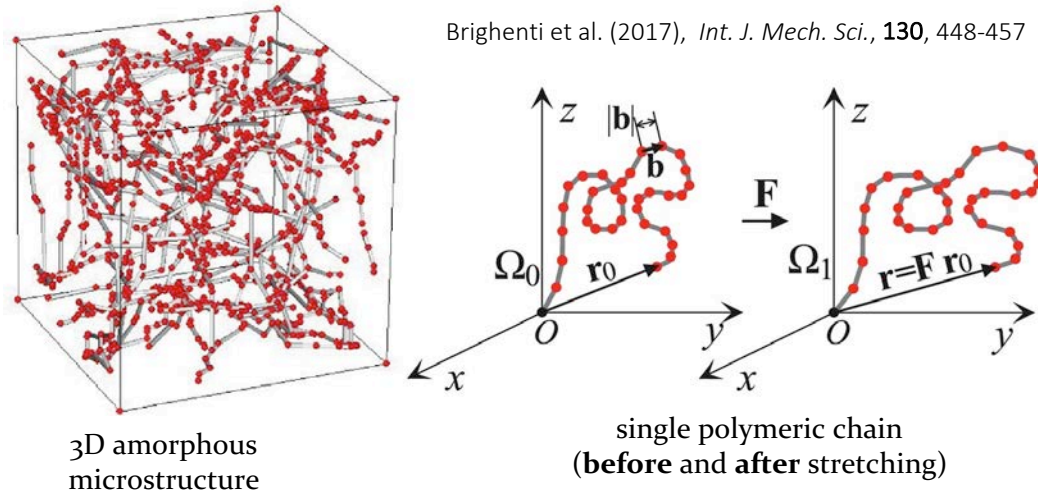
strain hardening

biphasic nature

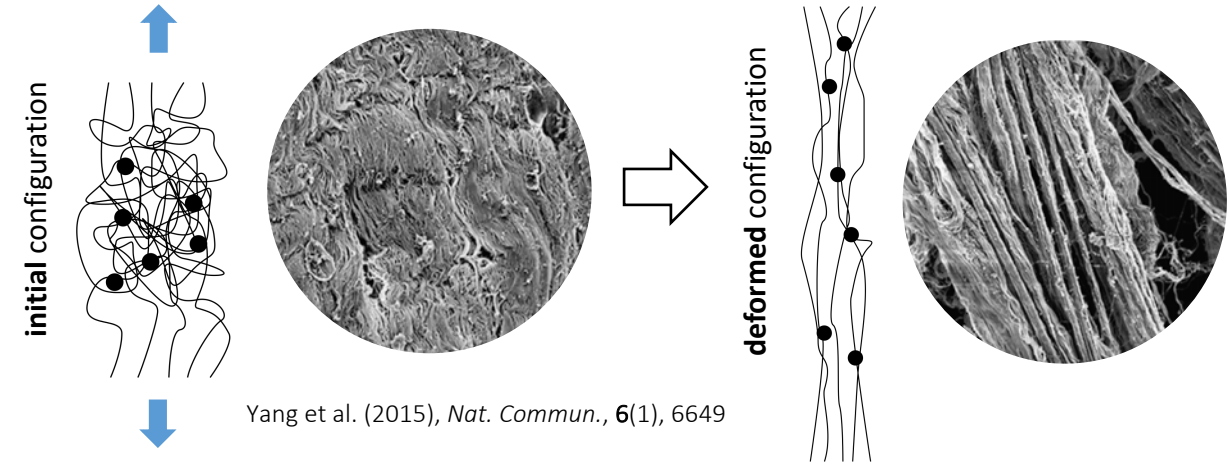
anisotropic behaviour

# Soft materials

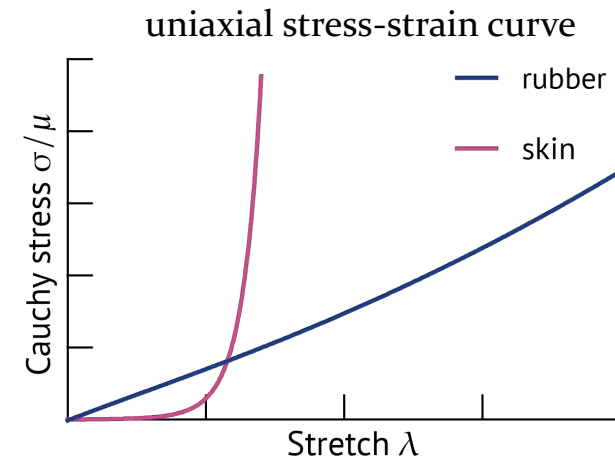
- The key feature of soft matter is **entropic elasticity**



## An example: the behaviour of skin



- the **entropy** of a single chain is derived from a Gaussian probability density function, as a function of the end-to-end distance  $r$
- elasticity arises through entropic **straightening** of polymeric chains, related to the variation of **entropy**
- by contrast, in **hard solids** ...
  - elasticity arises from variation of **internal energy** due to change in interatomic attractions (**energetic elasticity**)



- the dermis layer is made of **elastin** and **collagen**
- limited chain extensibility (**non-Gaussian** statistical theory)
- J-shaped** stress-strain curve

# Fracture process in soft matter

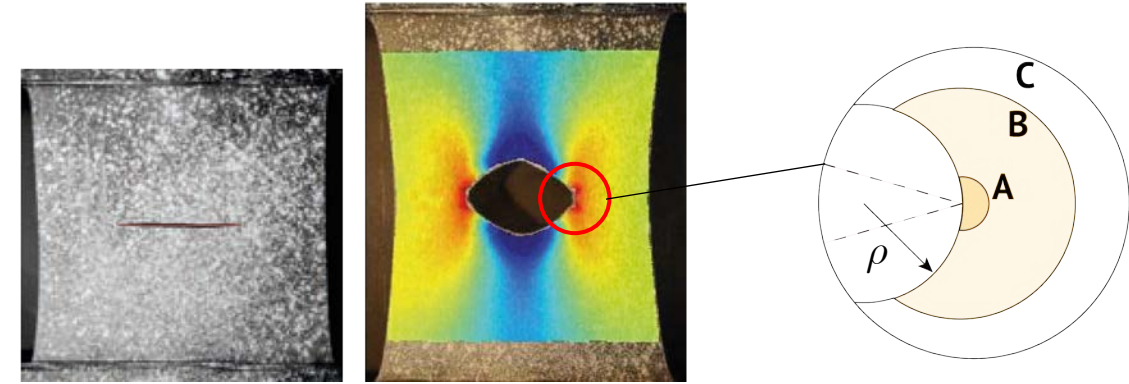
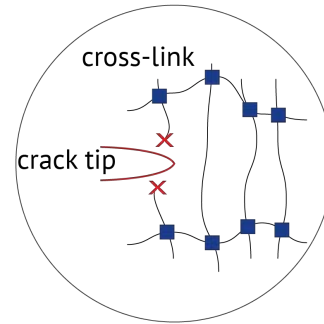
What happens when we **tear** soft materials?

Spagnoli, Terzano et al. (2019), *Appl. Sci.*, **9**(6), 1086

- at the **mesoscale**:
  - microscopic mechanisms of fracture include disentanglement by chain pull-out and bond rupture

Fracture energy  $\Gamma = nbN^{3/2}U_b$

monomer length  $\rightarrow$   $n$   
 bond energy  $\rightarrow$   $U_b$   
 density of chains  $\rightarrow$   $b$   
 number of monomers  $\rightarrow$   $N$

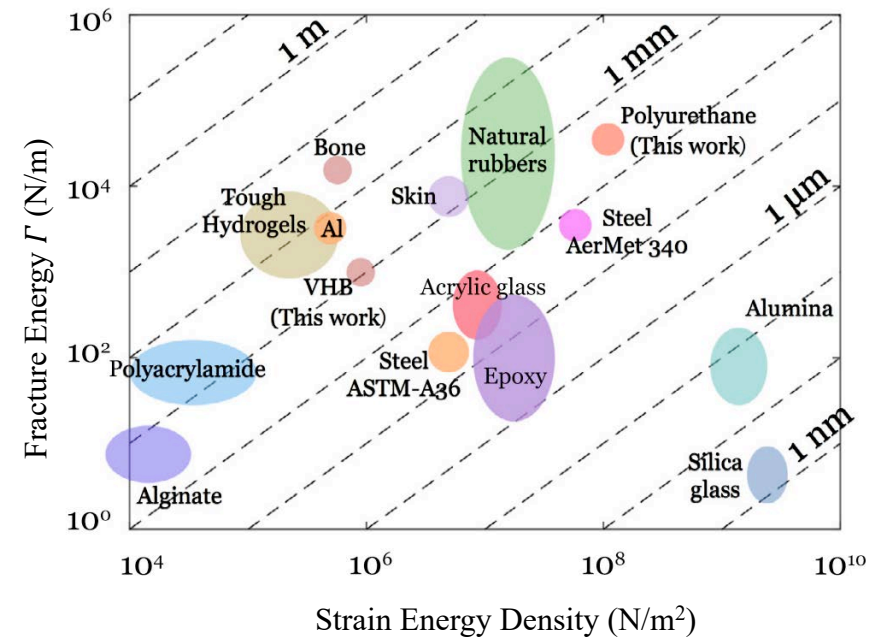


- at the **macroscale**:
  - crack blunting**: cracks are severely deformed upon loading
  - the **natural crack-tip radius** is the fundamental length scale defining soft materials by the point of view of fracture mechanics

natural crack-tip radius  $\rho \propto \frac{\Gamma}{W_c}$

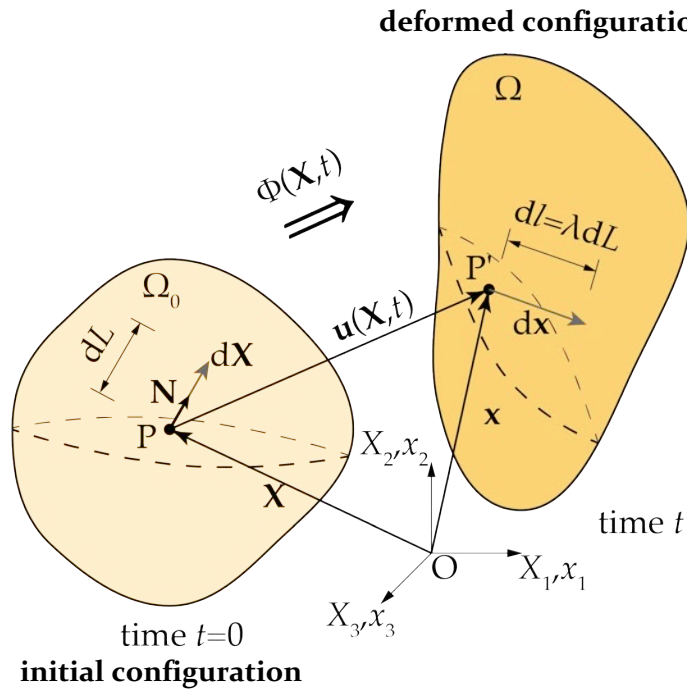
Strain energy density  $\rightarrow$   $W_c$

Chen et al. (2017), *Extrem. Mech. Lett.*, **10**, 50-57



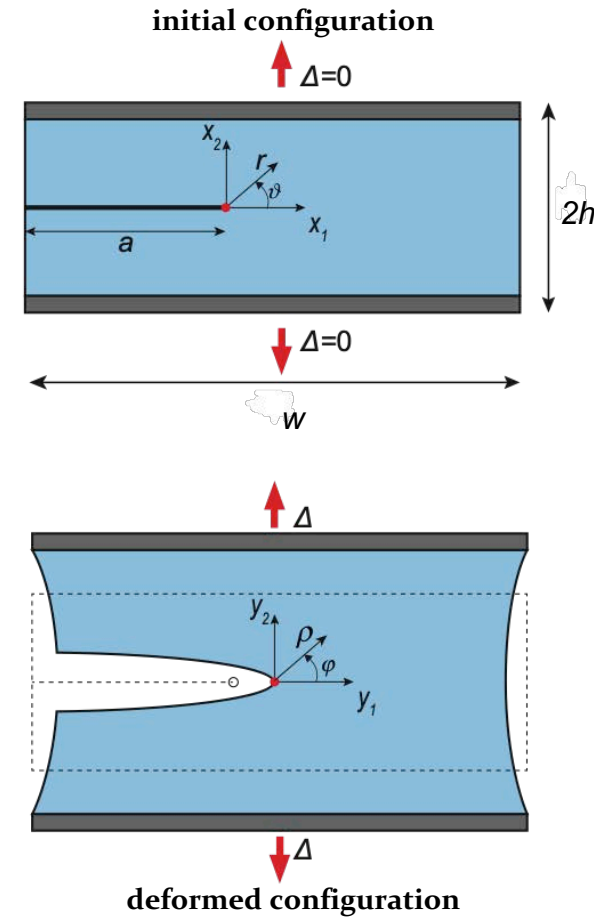
# Fracture in finite strain elasticity

## Analysis of the local crack-tip stresses in soft materials



current coordinates	$\mathbf{x} = \mathbf{X} + \mathbf{u}(\mathbf{X}, t)$
deformation gradient	$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$
left Cauchy-Green strain tensor	$\mathbf{b} = \mathbf{F}\mathbf{F}^T$
first strain invariant	$I_1 = \text{tr}\mathbf{b} = \lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2}$
Cauchy stress tensor	$\boldsymbol{\sigma} = -p\mathbf{I} + 2\frac{\partial W}{\partial I_1}\mathbf{b}$

neo-Hookean strain energy density	$W(I_1) = \frac{\mu}{2}(I_1 - 3)$
GNH strain energy density	$W(I_1) = \frac{\mu}{2b} \left\{ \left[ 1 + \frac{b}{n}(I_1 - 3) \right]^n - 1 \right\}$
exponential strain energy density (Fung model)	$W(I_1) = \frac{\mu}{2b} [\exp b(I_1 - 3) - 1]$



**hp:**  
isotropic hyperelastic incompressible behaviour  
pure shear geometry:

$$w, a \gg h \quad \mathbf{F} \sim \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda^{-1} \end{bmatrix}$$

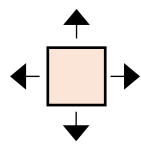
$$\lambda = 1 + \Delta/h$$

# Fracture in finite strain elasticity

## Analysis of the local crack-tip stresses in soft materials

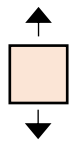
- In linear elastic materials

power of singularity



$$\left\{ \begin{aligned} \sigma_{11} &= \frac{K_I}{\sqrt{2\pi}} r^{-1/2} f_{11}(\vartheta) \\ \sigma_{22} &= \frac{K_I}{\sqrt{2\pi}} r^{-1/2} f_{22}(\vartheta) \\ \sigma_{12} &= \frac{K_I}{\sqrt{2\pi}} r^{-1/2} f_{12}(\vartheta) \end{aligned} \right.$$

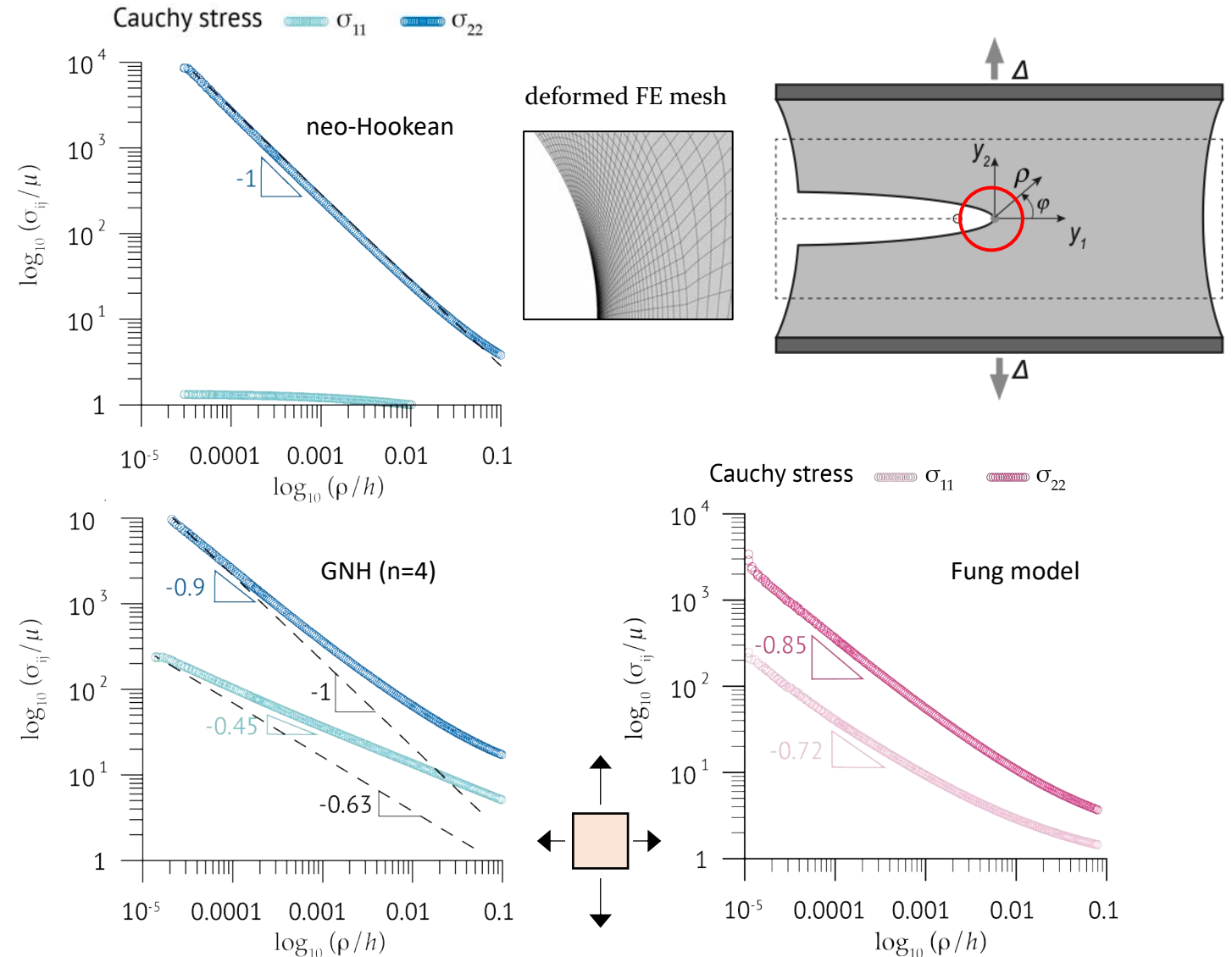
- In hyperelastic **neo-Hookean** materials



$$\left\{ \begin{aligned} \sigma_{11} &= \mu C_1^2 \\ \sigma_{22} &= \frac{\mu}{4} C_1 C_2^2 \rho^{-1} f_{22}(\rho, \varphi) \\ \sigma_{12} &= \mu C_1^{3/2} C_2 \rho^{-1/2} f_{12}(\rho, \varphi) \end{aligned} \right.$$

- In hyperelastic **strain-hardening** materials, the singularity depends on strain hardening

### Crack-tip stress fields from FE analysis



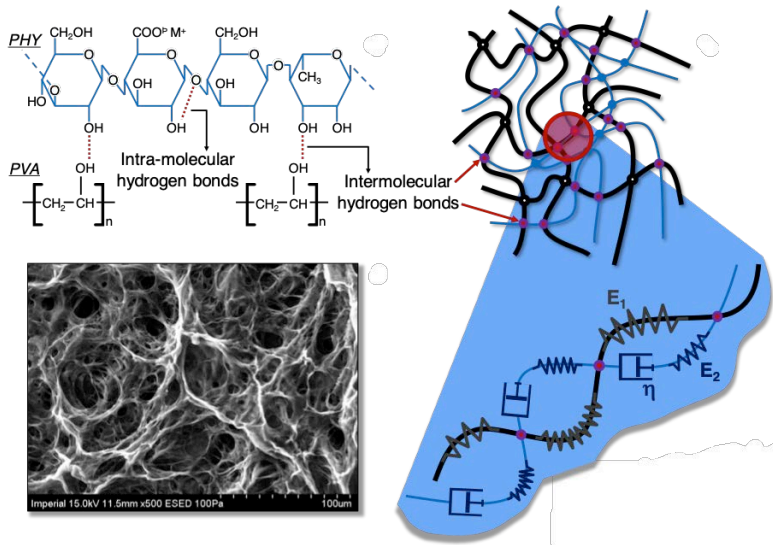
# Fracture in fluid-saturated soft materials

## Analysis of the fracture process in the brain tissue and mimicking hydrogels

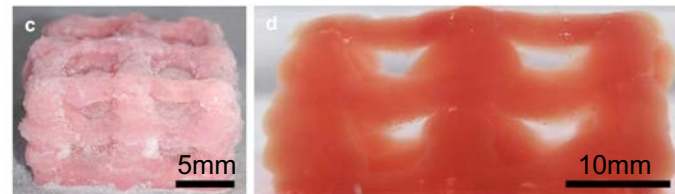
- the brain tissue is a **soft porous solid**, characterised by a very low stiffness ( $\sim 1$  kPa) and complex **time-dependent behaviour**
- hydrogels can be used as **synthetic mimicking materials**

composite hydrogel (CH)  
(poly-vinyl-Alcohol (PVA) + Phytigel (PHY))

⇒ can be obtained by **cryogenic 3D printing**



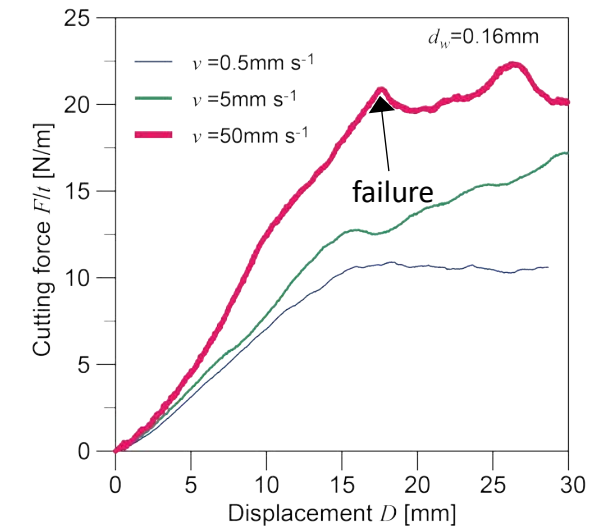
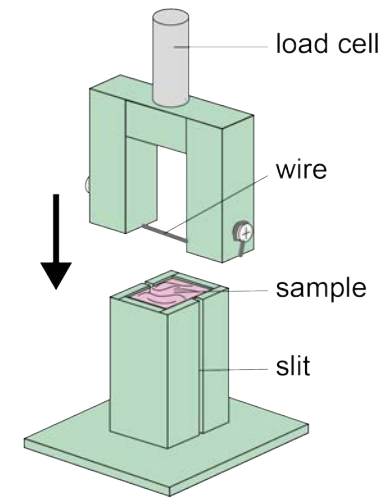
- extrusion-based** technique
- liquid to solid phase change of a hydrogel ink
- freeze-thaw cycle forms physical crosslinks
- resolution: 200  $\mu\text{m}$



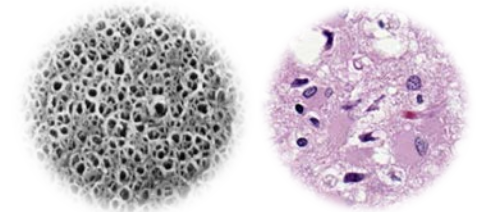
Forte et al. (2016), *Mater. Des.*, **112**, 227-236

Tan et al. (2017), *Sci. Rep.*, **7**(1), 16293

### Wire-cutting experiments (brain tissue)



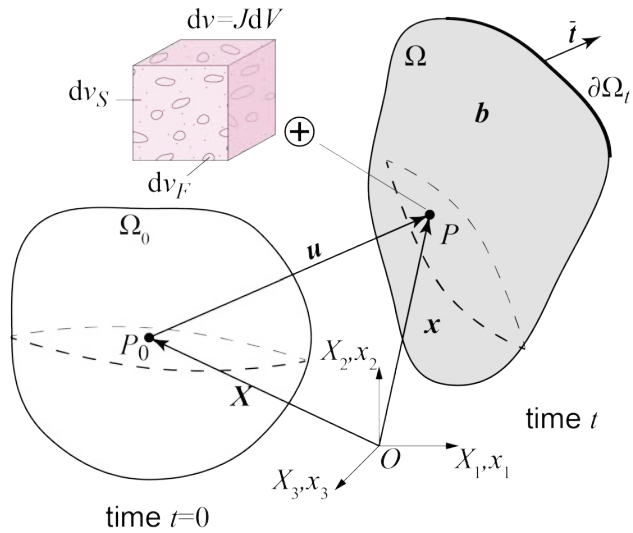
- we have employed wire-cutting experiments to test the **fracture properties** of:
  - porcine brain tissue
  - composite hydrogels
  - gelatine



Terzano, M., Spagnoli, A. *et al.*, submitted,  
"Fluid-solid interaction in the rate-dependent failure of  
brain tissue and biomimicking gels"

# Fracture in fluid-saturated soft materials

## Analysis of the fracture process in the brain tissue and mimicking hydrogels



**hp:** finite strain model  
hyperelastic compressible material  
incompressible viscous fluid (water)  
saturated conditions

modified principal stretch

$$\bar{\lambda}_i = J^{-1/3} \lambda_i$$

total and effective P-K stress

$$\mathbf{S} = \mathbf{S}' - J p_F \mathbf{C}^{-1} \quad S_i' = \frac{1}{\lambda_i} \frac{\partial W'(\bar{\lambda}_i)}{\partial \lambda_i}$$

Ogden's compressible model

$$W' = \sum_{i=1}^N \frac{2\mu_i}{\alpha_i^2} (\bar{\lambda}_1^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3) + \sum_{i=1}^N \frac{1}{D_i} (J - 1)^{2i}$$

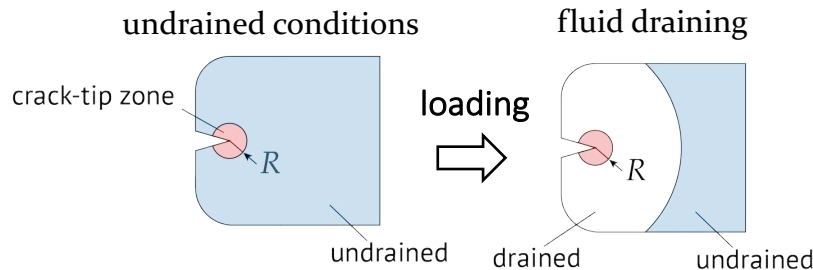
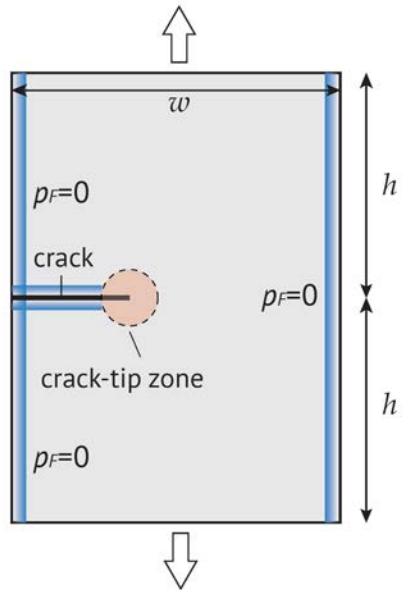
relative fluid velocity

$$\mathbf{w}(\mathbf{x}, t) = n_F (\dot{\mathbf{u}}_F - \dot{\mathbf{u}})$$

Darcy's law

$$J \mathbf{F}^{-1} \mathbf{w} = - \frac{k}{\gamma_F} \mathbf{I} \nabla p_F$$

Fluid pressure contours  $\lambda = \lambda_0$



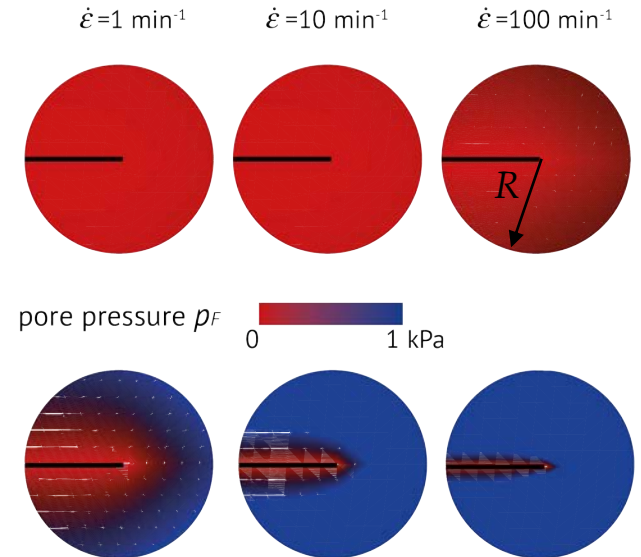
hydraulic conductivity (LT<sup>-1</sup>)

$$k = 1.25 \cdot 10^{-6} \text{ m/s}$$

gelatine 10% w/w

$$k = 1.25 \cdot 10^{-9} \text{ m/s}$$

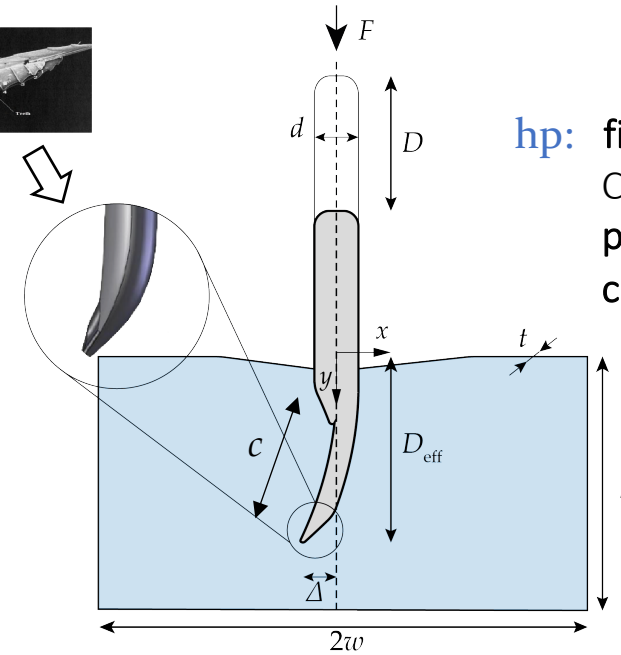
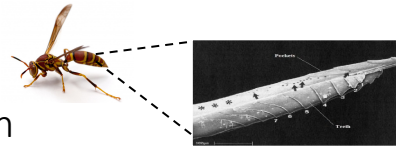
brain tissue





## Numerical simulation of curved insertion paths of needles in tissue mimicking hydrogels

- needle insertion can be treated as a problem of indentation cutting
- we consider a bio-inspired thin **flexible needle** with an **asymmetric tip** (programmable bevel-tipped needle, PBN)

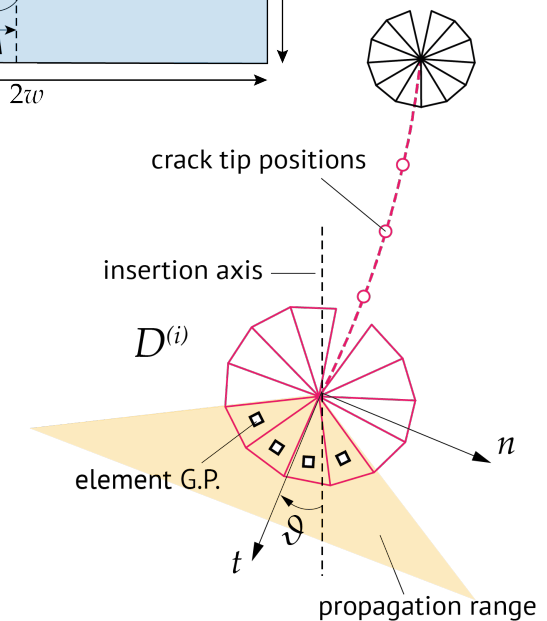
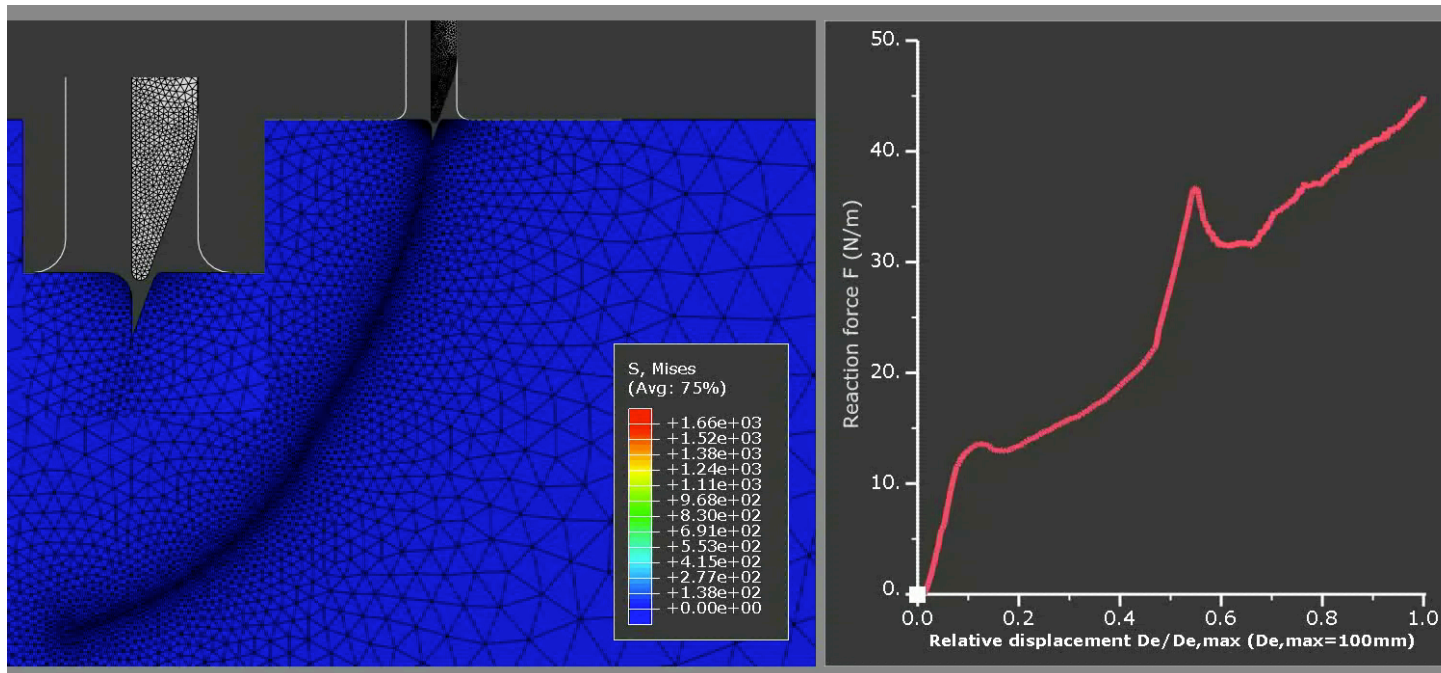


**hp:** finite strain elastic model  
Coulomb's friction  
plane strain conditions  
cohesive zone model

**KEY-POINT:** the propagation path is unknown in advance



iterative FE algorithm





Thank you for your attention

**Michele Terzano, PhD**  
Research Fellow  
Dept. of Engineering & Architecture, University of Parma  
[michele.terzano@unipr.it](mailto:michele.terzano@unipr.it)

### Aknowledgements

Prof. Andrea Spagnoli *PhD tutor, Università di Parma, Italy*  
Prof. Daniele Dini *PhD co-tutor, Imperial College, London, UK*  
Dr. Matthew Oldfield *University of Surrey, Surrey, UK*



Parma  
*Italian Capital of Culture 2020+21*